Online Appendix for "Conditional Investment-Cash Flow Sensitivities and Financing Constraints"

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1 Introduction

This appendix contains additional material for the study "Conditional Investment-Cash Flow Sensitivities and Financing Constraints," by Stephen R. Bond and Måns Söderbom; henceforth Bond and Söderbom (2011). Section 2 describes the numerical procedure used to generate the simulated investment data analyzed in the paper; Section 3 discusses additional simulation results.

2 The dynamic programming model

The firm's objective is to maximize the value of the firm:

$$
\begin{aligned}
V(K_t, B_{t-1}; a_t^P, a_t^T) \quad = \quad & \max_{I_t, N_t, B_t} \{\Pi\left(K_t, I_t; a_t^P, a_t^T\right) - \Phi(K_t, N_t) \\
& + B_t - (1 + i(K_t, B_{t-1}))B_{t-1} \\
& + \beta E_t\left[V\left((1-\delta)K_t + I_t, B_t; a_{t+1}^P, a_{t+1}^T\right)\right]\},
\end{aligned}
$$

subject to the following constraints (which hold for all t):

(i) Non-negative dividends:

$$
\Pi\left(K_t, I_t; a_t^P, a_t^T\right) - \Phi(K_t, N_t) + B_t - (1 + i(K_t, B_{t-1}))B_{t-1} + N_t \geq 0,
$$

(ii) Non-negative new equity:

$$N_t \geq 0$$

(iii) Non-negative debt:

$$B_t \geq 0$$

(iv) Capital evolution:

$$K_{t+1} = I_t + (1 - \delta) K_t$$

(v) Non-negative capital:

$$K_{t+1} \geq 0.$$

Exploiting linear homogeneity of the value function

Since the value function is linear homogeneous in capital, we can eliminate K_t from the state space by normalizing by K_t (see, for example, Bloom, 2009). Using the interest rate schedule,

$$i(K_t, B_{t-1}) = i + \eta \left(\frac{B_{t-1}}{K_t} \right),$$

the capital evolution constraint,

$$\frac{K_{t+1}}{K_t} = I_t/K_t - \delta + 1,$$

and defining $\pi_t = \Pi_t/K_t$, $v_t = V_t/K_t$, $\phi_t = \Phi_t/K_t$, $b_{t-1} = B_{t-1}/K_t$, $n_t = N_t/K_t$, and

$$\Psi_t = [I_t/K_t - \delta + 1] \, b_t - [1 + i + \eta b_{t-1}] \, b_{t-1}.$$

we can write the value of the firm as

$$\begin{aligned} v(b_{t-1}; a_t^P, a_t^T) \quad = \quad & \max_{I_t/K_t, n_t, b_t} \{ \pi \left(I_t/K_t; a_t^P, a_t^T \right) - \phi(n_t) \\ & + \Psi_t + [I_t/K_t - \delta + 1] \, \beta E_t \left[v \left(b_t; a_{t+1}^P, a_{t+1}^T \right) \right] \}, \end{aligned} \tag{A1}$$

subject to the following constraints (which hold for all t):

(i) Non-negative dividends:

$$\pi\left(I_t/K_t; a_t^P, a_t^T\right) - \phi(n_t) + \Psi_t + n_t \geq 0$$

(ii) Non-negative new equity:

$$n_t \geq 0$$

(iii) Non-negative debt:

$$b_t \geq 0$$

(iv) Non-negative capital:

$$I_t/K_t - \delta + 1 \geq 0.$$

Solving the Bellman equation using value function iteration

We solve the firm's optimization problem using value function iteration. The principles of our algorithm are as follows.

1. Start with a guess for the true value function $v(b_{t-1}; a_t^P, a_t^T)$. Call this guess v^1. Use it on the right-hand side of the Bellman equation (A1), and compute $E_t\left[v\left(b_t; a_{t+1}^P, a_{t+1}^T\right)\right]$ (more on how to compute this expectation below). Find the values of $\{I_t/K_t, n_t, b_t\}$ that maximize firm value subject to the relevant constraints.

2. Update the true value function $v(b_{t-1}; a_t^P, a_t^T)$ using the solution obtained in the previous step (i.e. this is the value v associated with optimal $\{I_t/K_t, n_t, b_t\}$). Call this updated guess v^2. Check if $v^2 = v^1$; if true, we have converged to the true function and so iteration can stop; if not set $j = 2$ and go to step 3.

3. Set $j = j + 1$. Use v^{j-1} on the right-hand side of the Bellman equation to compute $E_t\left[v\left(b_t; a^P_{t+1}, a^T_{t+1}\right)\right]$, and calculate the optimal choice rule $\{I_t/K_t, n_t, b_t\}$, subject to the relevant constraints. Update the value function, v^j. Check if $v^j = v^{j-1}$; If true, there is convergence and so iteration stops; if not, repeat step 3.

To implement this method we need to discretize the state and control space, and we need a way of calculating the expected value of the firm in $t+1$. We turn to these issues next.

State space and control space

We discretize the distribution of the debt variables b_{t-1} (state) and b_t (control) by constructing a finite set of permissible values $(b_1, b_2, ..., b_{M_b})$ where b_1 is the lowest permissible value, b_{M_b} the highest, and M_b is the number of permissible values. The results in the paper are based on models in which $b_1 = 0$, imposing non-negative borrowing (thus b_1 is determined by an economic constraint). The upper limit is set sufficiently high not to bind. Throughout we use $M_b = 18$.

We discretize the distribution of the investment variable (I_t/K_t) (control) by constructing a finite set of permissible values $(ik_1, ik_2, ..., ik_{M_i})$ where ik_1 is the lowest permissible value, ik_{M_i} the highest, and M_i is the number of permissible values. The upper and lower limits are set such that they will not bind. Throughout we use $M_i = 120$.

The distribution of new equity n_t is not discretized. Define

$$\Lambda_t = \left\{ \begin{array}{c} 0 \text{ if } \pi\left(I_t/K_t; a^P_t, a^T_t\right) + \Psi_t \geq 0 \\ 1 \text{ if } \pi\left(I_t/K_t; a^P_t, a^T_t\right) + \Psi_t < 0 \end{array} \right\}.$$

Thus $\Lambda_t = 1$ implies that the firm has to issue new equity to satisfy the non-negativity constraint for dividends. With the cost of issuing new equity given by $\phi(n_t) = 0.5\phi n_t^2$, optimal new equity n_t is as follows:

$$n_t = \left\{ \begin{array}{l} 0 \text{ if } \Lambda_t = 0 \\ -\left[\pi\left(I_t/K_t; a_t^P, a_t^T\right) - 0.5\phi n_t^2 + \Psi_t\right] > 0 \text{ if } \Lambda_t = 1 \end{array} \right\}$$

If $\phi > 0$, the firm will be financially constrained whenever $\Lambda_t = 1$, in which case new equity satisfies the quadratic

$$n_t + \pi\left(I_t/K_t; a_t^P, a_t^T\right) - 0.5\phi n_t^2 + \Psi_t = 0. \tag{A2}$$

Optimal new equity is the positive root of (A2):

$$n_t = \frac{-1 + \sqrt{1 - 2\phi\left[\pi\left(I_t/K_t; a_t^P, a_t^T\right) + \Psi_t\right]}}{\phi},$$

which is always real (since $\pi\left(I_t/K_t; a_t^P, a_t^T\right) + \Psi_t < 0$).

The serially uncorrelated shock a_t^T, assumed normally distributed in the model, is approximated using a discrete Gauss-Hermite quadrature. This implies the value function is evaluated at a finite set of variance dependent values $\sigma_T\sqrt{2}\left[\begin{array}{cccc} x_1 & x_2 & ... & x_{M_T}\end{array}\right]$ where the x_j are values (nodes) determined by the Gauss-Hermite quadrature, and M_T is the number of nodes. The expectation of the value function $E_t\left[v\left(b_t; a_{t+1}^P, a_{t+1}^T\right)\right]$ is computed by summing over the M_T possible realizations of the value function conditional on b_t and a_{t+1}^P using a set of weights determined by the Gauss-Hermite quadrature. See Judd (1998), pp.261-263 for details. Throughout we use $M_T = 3$.

The serially correlated component of the productivity parameter a^P is approximated using the method proposed by Tauchen (1986). The value function is evaluated at a finite set of possible realizations $\left(\bar{a}_1^P, \bar{a}_2^P, ..., \bar{a}_{M_P}^P\right)$, where M_P is the number of possible realizations. A transition matrix Γ is constructed, consisting of elements $\Gamma_{ij} = \Pr\left(a_{t+1}^P = \bar{a}_j^P | a_t^P = \bar{a}_i^P\right)$ determined by the persistence parameter ρ and the variance parameter σ_P. Having integrated over the distribution of a^T as described in the previous paragraph, we obtain the expectation $E_t\left[v\left(b_t; a^P_{t+1}, a^T_{t+1}\right)\right]$, conditional on the policy b_t and current state $\bar{a}^P_i$ by summing the value function over the M_P possible realizations of a^P_{t+1} using the elements in the ith column of the transition matrix Γ as weights. Throughout we set $M_P = 15$.

The simulated data

Once the value iteration has converged, we recover the policy functions for investment, debt and new equity. Based on these we simulate panel data according to the principles described in the text.

3 Additional simulation results

In this section we discuss additional simulation results. Table A.1 shows summary statistics for a simulated sample based on a model with no cost premium for external finance and no debt. Tables A.2-A.5 show results for the model without debt (cf. Table 1 in the main paper) with alternative values for the parameters in the productivity process. Table A.6 shows results for the model without debt with the variance of the measurement errors in investment reduced, resulting in a higher R-squared. Table A.7 shows estimates of the structural model with the non-negativity constraint on debt relaxed and the interest rate schedule defined as

$$i(K_{t+1}, B_t) = i + 1_{[B_t>0]}\eta\left(\frac{B_t}{K_{t+1}}\right).$$

3.1 Summary of the results

Table A.1 shows that, in the model with no cost premium for external finance and no debt, the mean of the average q variable is close to one, and the mean of the investment rates is close to 0.15, the rate of depreciation. The mean of the ratio of cash flow to capital is 0.187 which is close to the implied user cost of capital given our calibration. This in turn implies zero expected (excess) profits. Correlations between observed investment, average q, and cash-flow range between 0.35 and 0.71.

Table A.2 shows that reducing the serial correlation of productivity from 0.5 to 0.45 whilst keeping the relative importance of persistent and transitory components constant results in a reduction in the estimated cash-flow sensitivities for a given value of the parameter ϕ. For example, in the case where $\phi = 4$, the estimated coefficient on cash-flow falls from 0.100 (Table 1, column (iv), in the paper) to 0.047 (Table A.2, column (iv)). We still obtain a monotonic increase in the coefficient on cash-flow as issuing new equity becomes more expensive. Table A.3 shows that as the serial correlation of productivity is reduced further (to 0.30), the estimated cash-flow sensitivities are further reduced while the monotonicity result still holds.

In the models underlying Tables A.4 and A.5 we vary the parameter ρ, which measures the serial correlation in the persistent part of productivity, while keeping the serial correlation of overall productivity fixed at 0.5. This requires changes to the relative variance of persistent to overall productivity. The results in Table A.4, obtained for a model in which ρ is reduced from 0.8 to 0.7, indicate that a reduction ρ implies smaller estimated cash-flow sensitivities for a given value of the parameter ϕ. We still obtain a monotonic increase in the coefficient on cash-flow as issuing new equity becomes more expensive. In Table A.5, where we increase ρ to 0.81, we obtain higher estimated cash-flow sensitivities for a given value of the parameter ϕ than in our reference model shown in Table 1 in the paper.

Table A.6 shows results for the model without debt and with all the parameter values defined as specified in the paper, except that the variance of the measurement error term is reduced so as to increase the R-squared from 0.25 in the model with ϕ=0 to 0.6. The estimated coefficients on the cash-flow term are now equal to -0.001, 0.042, 0.071 and 0.102, for values of ϕ equal to 0, 1, 2 and 4, respectively. The estimated standard errors vary between 0.0037 and 0.0038.

Table A.7 shows results for the structural specification for models in which firms can choose negative debt and where negative debt is associated with a penalty as explained in Section 4.4 in the text (results for excess sensitivity specifications are shown in Table 5 in the paper). In all cases, the estimated coefficients are close to their theoretical counterparts.

References

[1] Bloom, N. (2009), "The impact of uncertainty shocks", *Econometrica*, 77:623-685.

[2] Bond, S.R. and M. Söderbom (2010), "Conditional investment-cash flow sensitivities and financing constraints", conditionally accepted, *Journal of the European Economic Association*.

[3] Judd, K.L. (1998), *Numerical Methods in Economics*, Cambridge:The MIT Press.

[4] Tauchen, G. (1986), "Finite state Markov-Chain approximations to univariate and vector autoregressions", *Economics Letters*, 20, 177-181.

Table A.1: Summary statistics. Model with no cost premium for external finance and no debt

	(i) Mean	(ii) Variance	(iii) Correlations		
			$\left(\frac{I}{K}\right)_t^{obs}$	Q_t	$\frac{C_t}{K_t}$
$\left(\frac{I}{K}\right)_t^{obs}$	0.163	0.06	1.00		
Q_t	1.064	0.15	0.50	1.00	
$\frac{C_t}{K_t}$	0.187	0.06	0.35	0.71	1.00

Sample size: N=2,000 T=14 Observations =28,000. These are the data underlying the regression results in Table 1, col. (i).

Table A.2: Model with costly equity & no debt; serial correlation of tfp reduced from 0.5 to 0.45 keeping the relative importance of persistent and transitory components constant (implies rho=0.72)

	(i)	(ii)	(iii)	(iv)
	ϕ=0	ϕ=1	ϕ=2	ϕ=4
Q	0.1971	0.1966	0.1968	0.1867
	0.0031	0.0031	0.0031	0.0031
C/K	0.0001	0.0172	0.0313	0.0473
	0.0051	0.0051	0.0051	0.0051
R-sq	0.2449	0.2613	0.272	0.2689

Note: Sample size: N=2000 T=14. Observations =28,000. OLS estimates.

Table A.3: Model with costly equity & no debt; serial correlation of tfp reduced from 0.5 to 0.3 keeping the relative importance of persistent and transitory components constant (implies rho = 0.48)

	(i)	(ii)	(iii)	(iv)
	ϕ=0	ϕ=1	ϕ=2	ϕ=4
Q	0.1959	0.2006	0.204	0.1979
	0.0031	0.0031	0.0032	0.0032
C/K	0	0.0103	0.0195	0.0288
	0.002	0.002	0.002	0.002
R-sq	0.2425	0.2749	0.299	0.3043

Note: Sample size: N=2000 T=14. Observations =28,000. OLS estimates.

Table A.4: Model with costly equity & no debt; rho reduced from 0.8 to 0.7 and the serial correlation of tfp kept fixed at 0.5

	(i)	(ii)	(iii)	(iv)
	ϕ=0	ϕ=1	ϕ=2	ϕ=4
Q	0.1972	0.1985	0.1995	0.1917
	0.0035	0.0035	0.0035	0.0035
C/K	-0.0001	0.0108	0.0204	0.0306
	0.0057	0.0057	0.0057	0.0057
R-sq	0.245	0.2602	0.2694	0.2656

Note: Sample size: N=2000 T=14. Observations =28,000. OLS estimates.

Table A.5: Model with costly equity & no debt; rho increased from 0.8 to 0.81 and the serial correlation of tfp kept fixed at 0.5

	(i)	(ii)	(iii)	(iv)
	ϕ=0	ϕ=1	ϕ=2	ϕ=4
Q	0.1974	0.1854	0.1787	0.1653
	0.0029	0.0029	0.003	0.003
C/K	-0.002	0.0497	0.0879	0.1182
	0.0084	0.0083	0.0084	0.0084
R-sq	0.2444	0.2478	0.2497	0.2422

Note: Sample size: N=2000 T=14. Observations =28,000. OLS estimates.

Table A.6: Model with costly equity & no debt; variance of the measurement errors increased so as to increase the R-sq in the first specification from 0.25 to 0.6

	(i)	(ii)	(iii)	(iv)
	ϕ=0	ϕ=1	ϕ=2	ϕ=4
Q	0.1989	0.1874	0.1801	0.1703
	0.0014	0.0014	0.0014	0.0014
C/K	-0.0008	0.0422	0.0705	0.1023
	0.0037	0.0037	0.0037	0.0038
R-sq	0.5972	0.6007	0.5971	0.5961

Note: Sample size: N=2000 T=14. Observations =28,000. OLS estimates.

Table A.7: Costly new equity, costly debt, firms can have negative debt: Structural model estimates

	(i) ϕ=0 η=.25 Coef.	s.e.	(ii) ϕ=1 η=.25 Coef.	s.e.	(iii) ϕ=2 η=1 Coef.	s.e.	(iv) ϕ=4 η=20 Coef.	s.e.
Constant	-0.0491	0.0022	-0.0456	0.0022	-0.0459	0.0022	-0.0510	0.0023
Q	0.1989	0.0021	0.1963	0.0020	0.1967	0.0021	0.2010	0.0021
(Q-b)*n	0.0345	0.0212	-0.1972	0.0525	-0.4128	0.0492	-0.8298	0.0735
R-sq	0.25		0.25		0.25		0.25	

Note: The table shows results for a model in which firms can have negative debt. See Section 4.4 in Bond & Söderbom (2010) for details. Sample size: N=2000 T=14. Observations =28,000. OLS estimates.