Applied Econometrics

Lecture 15:

Sample Selection Bias

Estimation of Nonlinear Models with Panel Data

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1. Introduction

In this the last lecture of the course we discuss two topics: How to estimate regressions if your sample is not random, in which case there may be sample selection bias; and how to estimate nonlinear models (focusing mostly on probit) if you have panel data.

References sample selection:

- Wooldridge (2002) Chapter 17.1-17.2; 17.4 (read carefully)
- François Bourguignon, Martin Fournier, Marc Gurgand "Selection Bias Corrections Based on the Multinomial Logit Model: Monte-Carlo Comparisons" DELTA working paper 2004-20, downloadable at http://www.delta.ens.fr/abstracts/wp200420.pdf (useful background reading for computer exercise 5)

References panel data models:

- Wooldridge (2002), Chapters 15.8.1-3; 16.8.1-2; 17.7 (read carefully).

2. Sample Selection

- Up to this point we have assumed the availability of a random sample from the underlying population. In practice, however, samples may not be random. In particular, samples are sometimes truncated by economic variables.
- We write our equation of interest (sometimes referred to as the 'structural equation' or the 'primary equation') as

\[ y_1 = x_1 \beta_1 + u_1, \]  

(2.1)
where \( x_1 \) is a vector of explanatory variables, all of which are exogenous in the population, and \( u_1 \) is an error term.

- Suppose selection is determined by the equation

\[
y_2 = \begin{cases} 
1 & \text{if } x_2 \delta_2 + v_2 \geq 0 \\
0 & \text{otherwise}
\end{cases},
\]  

(2.2)

where \( y_2 = 1 \) if we observe \( y_1 \) and zero otherwise, the vector \( x \) is assumed to contain all variables in the vector \( x_1 \) plus some more variables (unless otherwise stated), and \( v_2 \) is an error term. We assume we always observe \( x \), regardless of the value of \( y_2 \).

- **Example**: Suppose you want to study how education impacts on the wage an individual could earn in the labour market - i.e. the wage offer. Your plan is to run a regression in which log wage is the dependent variable and education is (let’s say) the only explanatory variable. You are primarily interested in the coefficient \( \beta_1 \) on education. Suppose in the population, education is uncorrelated with the residual \( u_1 \) - i.e. it is exogenous (this can be relaxed; more on this below). Thus, had you had access to a random sample, OLS would have been the best estimator.

- Suppose your sample contains a non-negligible proportion of individuals who do not work. For these individuals, there is no information on earnings, and so the corresponding observations cannot be used when estimating the wage equation. Thus you’re looking at having to estimate the earnings equation based on a non-random sample - what we shall refer to as a **selected sample**. Can the parameters of the wage offer equation - most importantly \( \beta_1 \) - be estimated without bias based on the selected sample?

- The general answer to that question is: It depends! Whenever we have a selected (non-random) sample, it is important to be clear on two things:

  - Circumstances under which OLS estimates, based on the selected sample, will suffer from bias
  - specifically **selectivity bias** - and circumstances when it won’t; and
– If there is selectivity bias in the OLS estimates: how to obtain estimates that are not biased by sample selection.

2.1. When will there be selection bias, and what can be done about it?

• I will now discuss estimation of the model above under the following assumptions:

• Assumption 17.1 (Wooldridge, p.562):

  – (a) \((x,y_2)\) are always observed, but \(y_1\) is only observed when \(y_2 = 1\) (sample selection);
  – (b) \((u_1,v_2)\) is independent of \(x\) with zero mean (\(x\) is exogenous in the population);
  – (c) \(v_2 \sim \text{Normal}(0,1)\) (distributional assumption); and
  – (d) \(E(u_1|v_2) = \gamma_2 v_2\) (residuals may be correlated; e.g. bivariate normality).

• Note that, given \(\text{var}(v_2) = 1\), \(\gamma_2\) measures the covariance between \(u_1\) and \(v_2\).

• The fundamental issue to consider when worrying about sample selection bias is why some individuals will not be included in the sample. As we shall see, sample selection bias can be viewed as a special case of endogeneity bias, arising when the selection process generates endogeneity in the selected sub-sample.

• In our model, and given assumption 17.1, sample selection bias arises when the residual in the selection equation (i.e. \(v_2\)) is correlated with the residual in the primary equation (i.e. \(u_1\)), i.e. whenever \(\gamma_2 \neq 0\). To see this, we will derive the expression for \(E(y_1|x, y_2 = 1)\), i.e. the expectation of the outcome variable conditional on observable \(x\) and selection into the sample.

• We begin by deriving \(E(y_1|x, v_2)\):

\[
E(y_1| x, v_2) = x_1 \beta_1 + E(u_1|x, v_2) \\
= x_1 \beta_1 + E(u_1|v_2) \\
E(y_1|x, v_2) = x_1 \beta_1 + \gamma_1 v_2.
\]
Part (b) of Assumption 17.1 (independence between $x$ and $u_1$) enables us to go from the first to the second line; part (d) enables us to go from the second to the third line.

- Since $v_2$ is not observable, eq (2.3) is not directly usable in applied work (since we can’t condition on unobservables when running a regression). To obtain an expression for the expected value of $y_1$ conditional on observables $x$ and the actual selection outcome $y_2$, we make use of the law of iterated expectations (see e.g. Wooldridge, p.19):

$$E(y_1|x, y_2) = E[E(y_1|x, v_2)|x, y_2].$$

Hence, using (2.3) we obtain

$$E(y_1|x, y_2) = E[(x_1\beta_1 + \gamma_1 v_2)|x, v_2, y_2],$$

$$E(y_1|x, y_2) = x_1\beta_1 + \gamma_1 E(v_2|x, y_2),$$

$$E(y_1|x, y_2) = x_1\beta_1 + \gamma_1 h(x, y_2),$$

where $h(x, y_2) = E(v_2|x, y_2)$ is some function (note that, since we condition on $x$ and $y_2$ it is not necessary to condition on $v_2$, hence the latter term vanishes when we go from the first to the second line).

- Because the selected sample has $y_2 = 1$, we only need to find $h(x, 1)$. Our model and assumptions imply

$$E(v_2|x, y_2 = 1) = E(v_2|v_2 \geq -x\delta_2),$$

and so we can use our ‘useful result’ appealed to in the previous lecture:

$$E(z|z > c) = \frac{\phi(c)}{1 - \Phi(c)}, \quad (2.4)$$

where $z$ follows a standard normal distribution, $c$ is a constant, $\phi$ denotes the standard normal
probability density function, and $\Phi$ is the standard normal cumulative density function. Thus

$$
E(v_2 | v_2 \geq -x \delta_2) = \frac{\phi(-x \delta_2)}{1 - \Phi(-x \delta_2)}
$$

$$
E(v_2 | v_2 \geq -x \delta_2) = \frac{\phi(x \delta_2)}{\Phi(x \delta_2)} = \lambda(x \delta_2),
$$

where $\lambda(\cdot)$ is the inverse Mills ratio (see Section 1 in the appendix for a derivation of the inverse Mills ratio). We now have a fully parametric expression for the expected value of $y_1$, conditional on observable variables $x$, and selection into the sample ($y_2 = 1$):

$$
E(y_1 | x, y_2 = 1) = x_1 \beta_1 + \gamma_1 \lambda(x \delta_2).
$$

### 2.1.1. Exogenous sample selection: $E(u_1 | v_2) = 0$

- Assume that the unobservables determining selection are independent of the unobservables determining the outcome variable of interest:

$$
E(u_1 | v_2) = 0.
$$

In this case, we say that sample selection is exogenous, and - here’s the good news - we can estimate the main equation of interest by means of OLS, since

$$
E(y_1 | x, y_2 = 1) = x_1 \beta_1,
$$

hence

$$
y_1 = x_1 \beta_1 + \zeta_i,
$$

where $\zeta_i$ is a mean-zero residual that is uncorrelated with $x_1$ in the selected sample (recall we assume exogeneity in the population). Examples:
- Suppose sample selection is randomized (or as good as randomized). Imagine an urn containing lots of balls, where 20% of the balls are red and 80% are black, and imagine participation in the sample depends on the draw from this urn: black ball, and you’re in; red ball and you’re not. In this case sample selection is independent of all other (observable and unobservable) factors (indeed \( \delta_2 = 0 \)). Sample selection is thus exogenous.

- Suppose the variables in the \( x \)-vector affect the likelihood of selection (i.e. \( \delta_2 \neq 0 \)). Hence individuals with certain observable characteristics are more likely to be included in the sample than others. Still, we’ve assumed \( x \) to be independent of the residual in the main equation, \( u_1 \), and so sample selection remains exogenous. In this case also - no problem.

### 2.1.2. Endogenous sample selection: \( E(u_1 \mid v_2) \neq 0 \)

Sample selection results in bias if the unobservables \( u_1 \) and \( v_2 \) are correlated, i.e. \( \gamma_1 \neq 0 \). Recall:

\[
E(y_1 \mid x, y_2 = 1) = x_1 \beta_1 + \gamma_1 \lambda(x \delta_2)
\]

- This equation tells us that the expected value of \( y_1 \), given \( x \) and observability of \( y_1 \) (i.e. \( y_2 = 1 \)) is equal to \( x_1 \beta_1 \), plus an additional term that depends on the inverse Mills ratio evaluated at \( z_i \gamma \).

Hence in the selected sample, actual \( y_1 \) is written as the sum of expected \( y_1 \) (conditional on \( x \) and selection) and a mean-zero residual:

\[
y_1 = x_1 \beta_1 + \gamma_1 \lambda(x \delta_2) + \varsigma_i,
\]

- It follows that if, based on the selected sample, we use OLS to run a regression in which \( y_1 \) is the dependent variable and \( x_1 \) is the set of explanatory variables, then \( \lambda(x \delta_2) \) will go into the residual; and to the extent that \( \lambda(x \delta_2) \) is correlated with \( x_1 \), the resulting estimates will be biased unless \( \gamma_1 = 0 \).
2.1.3. An example

Based on these insights, let’s now think about estimating the following simple wage equation based on a selected sample.

\[ \ln w_i = \beta_0 + \beta_1 \text{educ}_i + \varepsilon_i, \]

- Always when worrying about endogeneity, you need to be clear on the underlying mechanisms. So begin by asking yourself: What factors are likely to go into the residual \( \varepsilon_i \) in the wage equation? Clearly individuals with the same levels of education can obtain very different wages in the labour market, and given how we have written the model it follows by definition that the residual \( \varepsilon_i \) is the source of such wage differences. To keep the example simple, suppose I’ve convinced myself that the (true) residual \( \varepsilon_i \) consists of two parts:

\[ \varepsilon_i = \theta_1 m_i + e_i, \]

where \( m_i \) is personal ‘motivation’, which is unobserved (note!) and assumed uncorrelated with education in the population (clearly a debatable assumption, but let’s keep it simple), \( \theta_1 \) is a positive parameter, and \( e_i \) reflects the remaining source of variation in wages. Suppose for simplicity that \( e_i \) is independent of all variables except wages.

- I know from my econometrics textbook that there will be sample selection bias in the OLS estimator if the residual in the earnings equation \( \varepsilon_i \) is correlated with the residual in the selection equation. Let’s now relate this insight to economics, sticking to our example. Since motivation \( (m_i) \) is (assumed) the only economically interesting part of \( \varepsilon_i \), I thus need to ask myself: Is it reasonable to assume that motivation is uncorrelated with education in the selected sample? For now, maintain the assumption that motivation and education are uncorrelated in the population - hence had there been no sample selection, education would have been exogenous and OLS would have been fine.
• Still - and this is the key point - I may suspect that selection into the labour market depends on education and motivation:

\[
y_{2i} = \begin{cases} 
1 & \text{if } \gamma \cdot \text{educ}_i + (\theta_2m_i + \eta_i) \geq 0 \\
0 & \text{otherwise}
\end{cases},
\]

where \( \theta_2 \) is a positive parameter and \( \eta_i \) is a residual independent of all factors except selection. Because \( m_i \) is unobserved it will go into the residual, which will consist of the two terms inside the parentheses (\( . \)).

• The big question now is whether the factors determining selection are correlated with the wage residual \( \varepsilon_i = \theta_1m_i + e_i \). There are only three terms determining selection. Two of these are \( \eta_i \) and \( \text{educ}_i \), and they have been assumed uncorrelated with \( \varepsilon_i \). But what about motivation, \( m_i \)? Abstracting from the uninteresting case where \( \theta_1 \) and/or \( \theta_2 \) are equal to zero, we see that i) motivation determines selection; and ii) motivation is correlated with the wage residual since \( \varepsilon_i = \theta_1m_i + e_i \). So clearly we have endogenous selection.

• Does this imply that education is correlated with \( \varepsilon_i \) in the selected sample? Yes it does. The intuition as to why this is so is straightforward. Think about the characteristics (education and motivation) of the people that are included in the sample.

  – Someone with a low level of education must have a high level of motivation, otherwise he or she is likely not to be included in the sample (recall: the selection model implies that individuals with low levels of education and low levels of motivation are those most unlikely to be included in the sample).

  – In contrast, someone with a high level of education is fairly likely to participate in the labour market even if he or she happens to have a relatively low level of motivation.

• The implication is that, in the sample, the average level of motivation among those with little education will be higher than the average level of motivation with those with a lot of education. In
other words, education and motivation are negatively correlated in the sample, even though this
is not the case in the population.

- And since motivation goes into the residual (since we have no data on motivation - it’s unobserved),
it follows that education is (negatively) correlated with the residual in the selected sample. And
that’s why we get selectivity bias.

- Illustration: Figure 2 in the appendix.

2.2. How correct for sample selection bias?

I will now discuss the two most common ways of correcting for sample selection bias.

2.2.1. Method 1: Inclusion of control variables

The first method by which we can correct for selection bias is simple: include in the regression observed
variables that control for sample selection. In the wage example above , if we had data on motivation,
we could just augment the wage model with this variable:

$$\ln w_i = \beta_0 + \beta_1 educ_i + \theta_1 m_i + e_i.$$ 

More generally, recall that

$$E(y_1|x, v_2) = x_1 \beta_1 + \gamma_1 v_2.$$ 

and so if you have data on $v_2$, we could just use include this variable in the model as a control variable
for selection and estimate the primary equation using OLS. Such a strategy would completely solve the
sample selection problem.

Clearly this approach is only feasible if we have data on the relevant factors (e.g. motivation), which
may not always be the case. The second way of correcting for selectivity bias is to use the famous **Heckit
method**, developed by James Heckman in the 1970s.
2.2.2. Method 2: The Heckit method

We saw above that

$$E(y_1|x, y_2 = 1) = x_1\beta_1 + \gamma_1\lambda(x_2).$$

Using the same line of reasoning as for 'Method 1', it must be that if we had data on \(\lambda(x_2)\), we could simply add this variable to the model and estimate by OLS. Such an approach would be fine. Of course, in practice you would never have direct data on \(\lambda(x_2)\). However, the functional form \(\lambda(\cdot)\) is known and \(x\) is (it is assumed) observed. If so, the only missing piece is the parameter vector \(\delta_2\), which can be estimated by means of a probit model. The Heckit method thus consists of the following two steps:

1. Using all observations - those for which \(y_2\) is observed (selected observations) and those for which it is not - and estimate a probit model where \(y_2\) is the dependent variable and \(x\) are the explanatory variables. Based on the parameter estimates \(\hat{\delta}_2\), calculate the inverse Mills ratio for each observation:

$$\lambda(x_2) = \frac{\phi(x_2\hat{\delta}_2)}{\Phi(x_2\hat{\delta}_2)}.$$

2. Using the selected sample, i.e. all observations for which \(y_2\) is observed, and run an OLS regression in which \(y_2\) is the dependent variable and \(x_1\) and \(\lambda(x_2)\) are the explanatory variables:

$$y_1 = x_1\beta_1 + \gamma_1\lambda(x_2) + \zeta_i.$$

This will give consistent estimates of the parameter vector \(\beta_1\). That is, by including the inverse Mills ratio as an additional explanatory variable, we have corrected for sample selectivity.

Important considerations

- The Heckit procedure gives you an estimate of the parameter \(\gamma_1\), which measures the covariance between the two residuals \(u_1\) and \(v_2\). Under the null hypothesis that there is no selectivity bias, we have \(\gamma_1 = 0\). Hence testing \(H_0 : \gamma_1 = 0\) is of interest, and we can do this by means of a conventional
t-test. If you cannot reject $H_0 : \gamma_1 = 0$ then this indicates that sample selection does not result in significant bias, and so using OLS on the selected sample without including the inverse Mills ratio is fine - all this, under the assumption that the model is correctly specified and that (a)-(d) in Assumption 17.1 hold, of course.

- We assumed above that the vector $x$ (the determinants of selection) contains all variables that go into the vector $x_1$ (the explanatory variables in the primary equation), and possibly additional variables. In fact, it is highly desirable to specify the selection equation in such a way that there is at least one variable that determines selection, and which has no direct effect on $y_i$. In other words, it is important to impose at least one exclusion restriction. The reason is that if $x_1 = x$, the second stage of Heckit is likely to suffer from a collinearity problem, with very imprecise estimates as a result. Recall the form of the regression you run in the second stage of Heckit:

$$y_1 = x_1 \beta_1 + \gamma_1 \lambda \left( x \hat{\delta}_2 \right) + \varsigma_i.$$  

Clearly, if $x_1 = x$, then

$$y_1 = x \beta_1 + \gamma_1 \lambda \left( x \hat{\delta}_2 \right) + \varsigma_i.$$  

Remember that collinearity arises when one explanatory variable can be expressed as a linear function of one or several of the other explanatory variables in the model. In the above model $x_1$ enters linearly (the first term) and non-linearly (through inverse Mills ratio), which seems to suggest that there will not be perfect collinearity. However, if you look at the graph of the inverse Mills ratio (see Figure 1 in the appendix) you see that it is virtually linear over a wide range of values. Clearly had it been exactly linear there would be no way of estimating

$$y_1 = x_1 \beta_1 + \gamma_1 \lambda \left( x_1 \hat{\delta}_2 \right) + \varsigma_i.$$  

because $x_1$ would then be perfectly collinear with $\lambda \left( x_1 \hat{\delta}_2 \right)$. The fact that Mills ratio is virtually
linear over a wide range of values means that you can run into problems posed by severe (albeit not complete) collinearity. This problem is solved (or at least mitigated) if \( x \) contains one or several variables that are not included in \( x_1 \).

- Finally, always remember that in order to use the Heckit approach, you must have data on the explanatory variables for both selected and non-selected observations. This may not always be the case.

**Quantities of interest** Now consider partial effects. Suppose we are interested in the effects of changing the variable \( x_k \). It is useful to distinguish between three quantities of interest:

- The effect of a change on \( x_k \) on expected \( y_1 \) in the population:

\[
\frac{\partial E (y_1 | x_1 \beta_1)}{\partial x_k} = \beta_k
\]

For example, if \( x_k \) is education and \( y_1 \) is wage offer, then \( \beta_k \) measures the marginal effect of education on expected wage offer in the population.

- The effect of a change on \( x_k \) on expected \( y_1 \) for individuals in the population for whom \( y_1 \) is observed:

\[
\frac{\partial E (y_1 | x_1 \beta_1, y_2 = 1)}{\partial x_k} = \beta_k + \gamma_1 \frac{\partial \lambda \left( x_1 \delta_2 \right)}{\partial x_{ki}}.
\]

Recall that

\[
\lambda' (c) = -\lambda (c) [c + \lambda (c)],
\]

hence

\[
\frac{\partial E (y_1 | x_1 \beta_1, y_2 = 1)}{\partial x_k} = \beta_k - \delta_k \gamma_2 \lambda (x \delta_2) \left[ x \delta_2 + \lambda (x \delta_2) \right].
\]

It can be shown that \( c + \lambda (c) > 0 \), hence if \( \gamma_2 \) and \( \delta_k \) have the same sign, this partial effect is lower than that on expected \( y_1 \) in the population. In the context of education and wage offers, what is the intuition of this result? [Hint: increase education and less able individuals will work.]
• For a slightly modified version of the model, where \( y_1 = 0 \), rather than unobserved, if \( y_2 = 0 \), we might be interested in the effect of a change in \( x_k \) on \( E(y_1|x_1\beta_1) \) taking the zeros in \( y_1 \) into account. We have

\[
E(y_1|x_1\beta_1) = \Pr(y_2 = 1|x_2) \times E(y_1|x_1\beta_1, y_2 = 1) + \Pr(y_2 = 0|x_2) \times 0
\]

\[
E(y_1|x_1\beta_1) = \Phi(x_2) \times E(y_1|x_1\beta_1, y_2 = 1),
\]

and so "all" we need to do is find \( \frac{\partial E(y_1|x_1\beta_1)}{\partial x_k} \). This involves some tedious algebra, and so I will not go into detail. Check Cameron and Trivedi, *Microeconometrics: Methods & Applications*, p. 552 if you are interested.

**Estimation of Heckit in Stata**  
In Stata we can use the command `heckman` to obtain Heckit estimates. If the model is

\[
y_i = \beta_0 + \beta_1 x_1 i + u_i,
\]

\[
s_i = \begin{cases} 
1 & \text{if } \gamma_0 + \gamma_1 z_1 i + \gamma_2 x_1 i + v_i \geq 0 \\
0 & \text{otherwise} 
\end{cases},
\]

the syntax has the following form

```
heckman y x1, select (z1 x1) twostep
```

where the variable \( y \) is `missing` whenever an observation is not included in the selected sample. If you omit the `twostep` option you get full information maximum likelihood (FIML) estimates. Asymptotically, these two methods are equivalent, but in small samples the results can differ. Simulations have taught us that FIML is more efficient than the two-stage approach but also more sensitive to mis-specification due to, say, non-normal disturbance terms. In applied work it makes sense to consider both sets of results.

**EXAMPLES:** See Section 2.1-2.3 in appendix.
2.3. Extensions of the Heckit model

2.3.1. Endogenous explanatory variables

Now consider the case where \( x_1 \) contains a variable \( y_2 \) that is correlated with the error term \( u_i \). That is, \( y_2 \) is endogenous in the population. We write the population model as

\[
\begin{align*}
y_1 &= z_1 \delta_1 + \alpha_1 y_2 + u_1 \\
y_2 &= z_2 \delta_2 + v_2 \\
y_3 &= 1 [z_3 \delta_3 + v_3 \geq 0].
\end{align*}
\]

The first equation here is the structural equation of interest; the second equation is the reduced form equation for the endogenous explanatory variable \( y_2 \); and the third equation is the selectivity equation.

- Assumption 17.2: (a) \((z, y_3)\) always observed, \((y_1, y_2)\) observed when \( y_3 = 1 \) (sample selection); (b) \((u_1, v_3)\) is independent of \( z \) (\( z \) exogenous); (c) \( v_3 \sim \text{Normal}(0, 1) \) (distributional assumption); \( E(u_1|v_3) = \gamma_1 v_3 \) (residuals may be correlated; e.g. bivariate normality); (e) \( E(z'v_2 = 0) \), where \( z\delta_2 = z_1 \delta_{21} + z_2 \delta_{22}, \delta_{22} \neq 0 \) (valid and relevant instruments; exclusion restrictions)

Part (e) is new - instruments need to be orthogonal to the error term in the reduced form equation.

Note that the vector \( z_2 \) must contain at least two variables (at least one instrument for \( y_2 \), and at least one variable determining selection). Under these assumptions, estimation of the model parameters is relatively straightforward. We have

\[
\begin{align*}
y_1 &= z_1 \delta_1 + \alpha_1 y_2 + u_1 \\
y_1 &= z_1 \delta_1 + \alpha_1 y_2 + E(u_1|z,y_3) + e_1
\end{align*}
\]

in the population. Think of the term \( E[u_1|z,y_3] \) as the 'sample selection' term.
In the selected sample,

\[
E [u_{1} | z, y_{3} = 1] = E [v_{3} | z \delta_{3} + v_{3} \geq 0]
\]

\[
E [u_{1} | z, y_{3} = 1] = \gamma_{1} E [v_{3} | v_{3} \geq -z \delta_{3}]
\]

\[
E [u_{1} | z, y_{3} = 1] = \gamma_{1} \lambda (z \delta_{3}),
\]

and so

\[
y_{1} = z_{1} \delta_{1} + \alpha_{1} y_{2} + \gamma_{1} \lambda (z \delta_{3}) + e_{1}
\]

for the selected sample. This leads naturally to the following estimation recipe:

1. Obtain \( \tilde{\delta}_{3} \) by estimating the participation equation using a probit model. Construct \( \hat{\lambda}_{i3} = \lambda (z \hat{\delta}_{3}) \).

2. Using the selected sub-sample, estimate

\[
y_{1} = z_{i} \tilde{\delta}_{1} + \alpha_{1} y_{2} + \gamma_{1} \hat{\lambda}_{i3} + e_{1}
\]

using 2SLS, with instruments \((z, \hat{\lambda}_{i3})\).

Note that if we only have one exclusion restriction, predicted \( y_{2} \) will be (nearly) collinear with \( z_{1} \) and \( \hat{\lambda}_{i3} \). This is why we need at least two exclusion restrictions in the model.

EXAMPLE: See Section 2.4 in appendix.

2.3.2. Non-continuous outcome variables

We have focused on the case where \( y_{1} \), i.e. the outcome variable in the structural equation, is a continuous variable. However, sample selection models can be formulated for many different models - binary response models, censored models, duration models etc. The basic mechanism generating selection bias remains the same: correlation between the unobservables determining selection and the unobservables determining the outcome variable of interest.
Consider the following binary response model with sample selection:

\[
\begin{align*}
y_1 &= 1 \left[ x_1 \beta_1 + u_1 > 0 \right] \\
y_2 &= 1 \left[ x_2 \delta_2 + v_2 > 0 \right],
\end{align*}
\]

where \( y_1 \) is observed only if \( y_2 = 1 \), and \( x \) contains \( x_1 \) and at least one more variable. In this case, probit estimation of \( \beta_1 \) based on the selected sample will generally lead to inconsistent results, unless \( u_1 \) and \( v_2 \) are uncorrelated. Assuming that \( x \) is exogenous in the population (uncorrelated with \( u_1 \) and \( v_2 \)), we can use a two-stage procedure very similar to that discussed above:

1. Obtain \( \hat{\delta}_2 \) by estimating the participation equation using a probit model. Construct \( \hat{\lambda}_{i2} = \lambda \left( z_2 \hat{\delta}_2 \right) \).

2. Estimate the structural equation using probit, with \( \hat{\lambda}_{i2} \) added to the set of regressors:

\[ \Pr (y_1 | x_1, y_2 = 1) = \Phi \left( x_1 \beta_1 + \rho_1 \hat{\lambda}_{i2} \right), \]

where \( \rho_1 \) measures the correlation between the residuals \( u_1 \) and \( v_2 \) (note: correlation will be the same as the covariance, due to unity variance for the two residuals).

This is a good procedure for testing the null hypothesis that there is no selection bias (in which case \( \rho_1 = 0 \)). If, based on this test we decide there is endogenous selection, we might choose to estimate the two equations of the model simultaneously (in Stata: heckprob). This produces the right standard errors, and recovers the structural parameters \( \beta_1 \) rather than a scaled version of this vector.

### 2.3.3. Non-binary selection equation

Alternatively, it could be that the selection equation is not a binary response model - see Section IV in Vella (1999) for an overview if you are interested. In computer exercise 5 we will study the case where selection is modelled by means of a multinomial logit. An excellent survey paper in this context is that by Bourguignon, Fournier and Gurgand. Please have a look at this paper before the computer lab.
on Friday.

3. Estimation of Nonlinear Models with Panel Data

I will now discuss how probit, logit, tobit and heckit can be estimated when panel data are available. I will focus on non-dynamic models and mostly on the binary choice models.\(^1\)

3.1. Binary choice models for panel data

Using a latent variable framework, we write the panel binary choice model as

\[
y_{it}^* = x_{it}\beta + c_i + u_{it},
\]

\[
y_{it} = 1[y_{it}^* > 0],
\]

and

\[
Pr(y_{it} = 1|x_{it}, c_i) = G(x_{it}\beta + c_i),
\]

where \(G(.)\) is either the standard normal CDF (probit) or the logistic CDF (logit).

- Recall that, in linear models, it is easy to eliminate \(c_i\) by means of first differencing or using within transformation.

- Those routes are not open to us here, unfortunately, since the model is nonlinear (e.g. differencing equation (3.1) does not remove \(c_i\)).

- Moreover, if we attempt to estimate \(c_i\) directly by adding \(N - 1\) individual dummy variables to the probit or logit specification, this will result in severely biased estimates of \(\beta\) unless \(T\) is large.

This is known as the **incidental parameters problem**: with \(T\) small, the estimates of the \(c_i\)

---

\(^1\)As you know, including a lagged dependent variable in the set of explanatory variables complicates the estimation of standard linear panel data models. Conceptually similar problems arise for nonlinear models. Consider a dynamic probit model for example:

\[
Pr(y_{it} = 1|x_{it}, c_i) = \Phi(y_{it-1} + z_{it}\delta + c_i),
\]

The methods discussed below are generally not well suited for estimating such a model. If you are interested, check out http://www.soderbom.net/binarychoice2.pdf for a discussion.
are inconsistent (i.e. increasing \( N \) does not remove the bias), and, unlike the linear model, the inconsistency in \( c_i \) has a 'knock-on effect' in the sense that the estimate of \( \beta \) becomes inconsistent too!

3.1.1. Incidental parameters: An example

Consider the logit model in which \( T = 2 \), \( \beta \) is a scalar, and \( x_{it} \) is a time dummy such that \( x_{i1} = 0, x_{i2} = 1 \). Thus

\[
\Pr (y_{it} = 1|x_{i1}, c_i) = \frac{\exp (\beta \cdot 0 + c_i)}{1 + \exp (\beta \cdot 0 + c_i)} \equiv \Lambda (\beta \cdot 0 + c_i),
\]

\[
\Pr (y_{it} = 1|x_{i2}, c_i) = \frac{\exp (\beta \cdot 1 + c_i)}{1 + \exp (\beta \cdot 1 + c_i)} \equiv \Lambda (\beta \cdot 1 + c_i).
\]

Suppose we attempt to estimate this model with \( N \) dummy variables included to control for the individual effects. There would thus be \( N+1 \) parameters in the model: \( c_1, c_2, \ldots, c_i, \ldots c_N, \beta \). Our parameter of interest is \( \beta \).

However, it can be shown that, in this particular case,

\[
p \lim_{N \to \infty} \hat{\beta} = 2\beta.
\]

That is, the probability limit of the logit dummy variable estimator - for this admittedly very special case - is double the true value of \( \beta \). With a bias of 100% in very large (infinite) samples (with respect to \( N \)), this is not a very useful approach. This form of inconsistency also holds in more general cases: unless \( T \) is large, the logit dummy variable estimator will not work.

- So how can we proceed? I will discuss three common approaches: the traditional random effects (RE) probit (or logit) model; the conditional fixed effects logit model; and the Mundlak-Chamberlain approach.
3.1.2. The traditional random effects (RE) probit

Model:

\[ y_{it}^* = x_{it}\beta + c_i + u_{it}, \]
\[ y_{it} = 1[y_{it}^* > 0], \]

and

\[ \Pr (y_{it} = 1|x_{it}, c_i) = G(x_{it}\beta + c_i), \]

Assumptions:

- \( c_i \) and \( x_{it} \) are independent
- \( x_{it} \) are strictly exogenous (this will be necessary for it to be possible to write the likelihood of observing a given series of outcomes as the product of individual likelihoods).
- \( c_i \) has a normal distribution with zero mean and variance \( \sigma^2 \) (note: homoskedasticity).
- \( y_{i1}, ..., y_{iT} \) are independent conditional on \( (x_i, c_i) \) - this rules out serial correlation in \( y_{it} \), conditional on \( (x_i, c_i) \). This assumption enables us to write the likelihood of observing a given series of outcomes as the product of individual likelihoods. The assumption can easily be relaxed - see eq. (15.68) in Wooldridge (2002).
- Clearly these are restrictive assumptions, especially since endogeneity in the explanatory variables is ruled out. The only advantage (which may strike you as rather marginal) over a simple pooled probit model is that the RE model allows for serial correlation in the unobserved factors determining \( y_{it} \), i.e. in \( (c_i + u_{it}) \).
- However, it is fairly straightforward to extend the model and allow for correlation between \( c_i \) and \( x_{it} \) - this is precisely what the Mundlak-Chamberlain approach achieves, as we shall see below.
Clearly, if $c_i$ had been observed, the likelihood of observing individual $i$ would have been

$$\prod_{t=1}^{T} \left[ \Phi (\mathbf{x}_{it}\beta + c_i) \right]^{y_{it}} \left[ 1 - \Phi (\mathbf{x}_{it}\beta + c_i) \right]^{(1-y_{it})},$$

and it would have been straightforward to maximize the sample likelihood conditional on $\mathbf{x}_{it}, c_i, y_{it}$.

Because the $c_i$ are unobserved, however, they cannot be conditioned on in the likelihood function. As discussed above, a dummy variables approach cannot be used, unless $T$ is large. What can we do?

Recall from basic statistics (Bayes’ theorem for probability densities) that, in general,

$$f_{x|y}(x,y) = \frac{f_{xy}(x,y)}{f_y(y)},$$

where $f_{x|y}(x,y)$ is the conditional density of $X$ given $Y = y$; $f_{xy}(x,y)$ is the joint distribution of random variables $X, Y$; and $f_y(y)$ is the marginal density of $Y$. Thus,

$$f_{xy}(x,y) = f_{x|y}(x,y) f_y(y).$$

Moreover, the marginal density of $X$ can be obtained by integrating out $y$ from the joint density

$$f_x(x) = \int f_{xy}(x,y) dy = \int f_{x|y}(x,y) f_y(y) dy.$$

Clearly we can think about $f_x(x)$ as a likelihood contribution. For a linear model, for example, we might write

$$f_x(\varepsilon) = \int f_{x|c}(\varepsilon,c) dc = \int f_{x|c}(\varepsilon,c) f_c(c) dc,$$

where $\varepsilon_{it} = y_{it} - (\mathbf{x}_{it}\beta + c_i)$.  

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In the context of the traditional RE probit, we integrate out \( c_i \) from the likelihood as follows:

\[
L_i \left( y_{i1}, \ldots, y_{iT} | x_{i1}, \ldots, x_{iT}; \beta, \sigma_c^2 \right) = \\
\int \prod_{t=1}^{T} \left[ \Phi \left( x_{it} \beta + c \right)^{y_{it}} \left[ 1 - \Phi \left( x_{it} \beta + c \right) \right]^{\left( 1 - y_{it} \right)} \left[ 1 / \sigma_c \right] \phi \left( c / \sigma_c \right) dc. \right]
\]

In general, there is no analytical solution here, and so numerical methods have to be used. The most common approach is to use a Gauss-Hermite quadrature method, which amounts to approximating

\[
\int \prod_{t=1}^{T} \left[ \Phi \left( x_{it} \beta + c \right)^{y_{it}} \left[ 1 - \Phi \left( x_{it} \beta + c \right) \right]^{\left( 1 - y_{it} \right)} \left[ 1 / \sigma_c \right] \phi \left( c / \sigma_c \right) dc \\
\]

as

\[
\pi^{-1/2} \sum_{m=1}^{M} w_m \prod_{i=1}^{T} \left[ \Phi \left( x_{it} \beta + \sqrt{2} \sigma_c g_m \right) \right]^{y_{it}} \left[ 1 - \Phi \left( x_{it} \beta + \sqrt{2} \sigma_c g_m \right) \right]^{\left( 1 - y_{it} \right)}, \quad (3.2)
\]

where \( M \) is the number of nodes, \( w_m \) is a prespecified weight, and \( g_m \) a prespecified node (prespecified in such a way as to provide as good an approximation as possible of the normal distribution).

For example, if \( M = 3 \), we have

\[
\begin{array}{cc}
  w_m & g_m \\
  0.2954 & -1.2247 \\
  1.1826 & 0.0000 \\
  0.2954 & 1.2247 \\
\end{array}
\]
in which case (3.2) can be written out as

\[
0.1667 \prod_{t=1}^{T} \left[ \Phi\left( \mathbf{x}_{it}\mathbf{\beta} - 1.731\sigma_c \right) \right]^{y_{it}} \left[ 1 - \Phi\left( \mathbf{x}_{it}\mathbf{\beta} - 1.731\sigma_c \right) \right]^{(1 - y_{it})} \\
+ 0.6667 \prod_{t=1}^{T} \left[ \Phi\left( \mathbf{x}_{it}\mathbf{\beta} \right) \right]^{y_{it}} \left[ 1 - \Phi\left( \mathbf{x}_{it}\mathbf{\beta} \right) \right]^{(1 - y_{it})} \\
+ 0.1667 \prod_{t=1}^{T} \left[ \Phi\left( \mathbf{x}_{it}\mathbf{\beta} + 1.731\sigma_c \right) \right]^{y_{it}} \left[ 1 - \Phi\left( \mathbf{x}_{it}\mathbf{\beta} + 1.731\sigma_c \right) \right]^{(1 - y_{it})}.
\]

In practice a larger number of nodes than 3 would of course be used (the default in Stata is \( M = 12 \)). Lists of weights and nodes for given values of \( M \) can be found in the literature.

- To form the sample log likelihood, we simply compute weighted sums in this fashion for each individual in the sample, and then add up all the individual likelihoods expressed in natural logarithms:

\[
\log L = \sum_{i=1}^{N} \log L_i \left( y_{i1}, ..., y_{iT} | \mathbf{x}_{i1}, ..., \mathbf{x}_{iT}; \mathbf{\beta}, \sigma_c^2 \right).
\]

Marginal effects at \( c_i = 0 \) can be computed using standard techniques. This model can be estimated in Stata using the \texttt{xtprobit} command.

- EXAMPLE: Modelling exports in Ghana using probit and allowing for unobserved individual effects. Appendix Section 3.1

Whilst perhaps elegant, the above model does not allow for a correlation between \( c_i \) and the explanatory variables, and so does not achieve anything in terms of addressing an endogeneity problem. We now turn to more useful models in that context.

3.1.3. The "fixed effects" logit model

Now return to the panel logit model:

\[
\Pr \left( y_{it} = 1 | \mathbf{x}_{it}, c_i \right) = \Lambda \left( \mathbf{x}_{it}\mathbf{\beta} + c_i \right).
\]
One important advantage of this model over the probit model is that will be possible to obtain a consistent estimator of $\beta$ without making any assumptions about how $c_i$ is related to $x_{it}$ (however, you need strict exogeneity to hold; cf. within estimator for linear models).

This is possible, because the logit functional form enables us to eliminate $c_i$ from the estimating equation, once we condition on what is sometimes referred to as a "minimum sufficient statistic" for $c_i$.

To see this, assume $T = 2$, and consider the following conditional probabilities:

$$\Pr (y_{i1} = 0, y_{i2} = 1|x_{i1}, x_{i2}, c_i, y_{i1} + y_{i2} = 1),$$

and

$$\Pr (y_{i1} = 1, y_{i2} = 0|x_{i1}, x_{i2}, c_i, y_{i1} + y_{i2} = 1).$$

The key thing to note here is that we condition on $y_{i1} + y_{i2} = 1$, i.e. that $y_{it}$ changes between the two time periods. For the logit functional form, we have

$$\Pr (y_{i1} + y_{i2} = 1|x_{i1}, x_{i2}, c_i) = \frac{\exp (x_{i1}\beta + c_i)}{1 + \exp (x_{i1}\beta + c_i)} \frac{1}{1 + \exp (x_{i2}\beta + c_i)} \frac{1}{1 + \exp (x_{i2}\beta + c_i)},$$

or simply

$$\Pr (y_{i1} + y_{i2} = 1|x_{i1}, x_{i2}, c_i) = \frac{\exp (x_{i1}\beta + c_i) + \exp (x_{i2}\beta + c_i)}{[1 + \exp (x_{i1}\beta + c_i)] [1 + \exp (x_{i2}\beta + c_i)].}$$

Furthermore,

$$\Pr (y_{i1} = 0, y_{i2} = 1|x_{i1}, x_{i2}, c_i) = \frac{1}{1 + \exp (x_{i1}\beta + c_i)} \frac{\exp (x_{i2}\beta + c_i)}{1 + \exp (x_{i2}\beta + c_i)}. $$
hence, conditional on \( y_{i1} + y_{i2} = 1 \),

\[
\Pr (y_{i1} = 0, y_{i2} = 1 | x_{i1}, x_{i2}, c_i, y_{i1} + y_{i2} = 1) = \frac{\exp (x_{i2} \beta + c_i)}{\exp (x_{i1} \beta + c_i) + \exp (x_{i2} \beta + c_i)},
\]

or

\[
\Pr (y_{i1} = 0, y_{i2} = 1 | x_{i1}, x_{i2}, y_{i1} + y_{i2} = 1) = \frac{\exp (\Delta x_{i2} \beta)}{1 + \exp (\Delta x_{i2} \beta)}
\]

- The key result here is that the \( c_i \) are eliminated. It follows that

\[
\Pr (y_{i1} = 1, y_{i2} = 0 | x_{i1}, x_{i2}, y_{i1} + y_{i2} = 1) = \frac{1}{1 + \exp (\Delta x_{i2} \beta)}.
\]

- Remember:

1. These probabilities condition on \( y_{i1} + y_{i2} = 1 \)
2. These probabilities are independent of \( c_i \).

Hence, by maximizing the following **conditional** log likelihood function

\[
\log L = \sum_{i=1}^{N} \left\{ d_{01i} \ln \left( \frac{\exp (\Delta x_{i2} \beta)}{1 + \exp (\Delta x_{i2} \beta)} \right) + d_{10i} \ln \left( \frac{1}{1 + \exp (\Delta x_{i2} \beta)} \right) \right\},
\]

we obtain consistent estimates of \( \beta \), regardless of whether \( c_i \) and \( x_{iit} \) are correlated.

- The trick is thus to condition the likelihood on the outcome series \((y_{i1}, y_{i2})\), and in the more general case \((y_{i1}, y_{i2}, ..., y_{iT})\). For example, if \( T = 3 \), we can condition on \( \sum_{t} y_{it} = 1 \), with possible sequences \( \{1, 0, 0\}, \{0, 1, 0\} \) and \( \{0, 0, 1\} \), or on \( \sum_{t} y_{it} = 2 \), with possible sequences \( \{1, 1, 0\}, \{1, 0, 1\} \) and \( \{0, 1, 1\} \). Stata does this for us, of course. This estimator is requested in Stata by using **xtlogit** with the **fe** option.

**EXAMPLE:** Exports in Ghana using FE logit. Appendix Section 3.2
Note that the logit functional form is crucial for it to be possible to eliminate the \( c_i \) in this fashion. It won’t be possible with probit. So this approach is not really very general. Another awkward issue concerns the interpretation of the results. The estimation procedure just outlined implies we do not obtain estimates of \( c_i \), which means we can’t compute marginal effects.

### 3.1.4. Modelling the random effect as a function of x-variables

The previous two methods are useful, but arguably they don’t quite help you achieve enough:

- the traditional random effects probit/logit model requires strict exogeneity and zero correlation between the explanatory variables and \( c_i \);
- the fixed effects logit relaxes the latter assumption but we can’t obtain consistent estimates of \( c_i \) and hence we can’t compute the conventional marginal effects in general.

We will now discuss an approach which, in some ways, can be thought of as representing a middle way. Start from the latent variable model

\[
y_{it}^{*} = x_{it}\beta + c_i + e_{it}, \]

\[
y_{it} = 1_{[y_{it}^{*}>0]}.
\]

Consider writing the \( c_i \) as an **explicit function** of the x-variables, for example as follows:

\[
c_i = \psi + \bar{x}_i\xi + a_i \tag{3.3}
\]

or

\[
c_i = \phi + x_i\tau + b_i \tag{3.4}
\]

where \( \bar{x}_i \) is an average of \( x_{it} \) over time for individual \( i \) (hence time invariant); \( x_i \) contains \( x_{it} \) for all \( t \); \( a_i \) is assumed uncorrelated with \( \bar{x}_i \); \( b_i \) is assumed uncorrelated with \( x_i \). Equation (3.3) is easier to
implement and so we will focus on this (see Wooldridge, 2002, pp. 489-90 for a discussion of the more general specification).

- Assume that \( \text{var}(a_i) = \sigma_a^2 \) is constant (i.e. there is homoskedasticity) and that \( e_i \) is normally distributed - the model that then results is known as **Chamberlain’s random effects probit model**. You might say (3.3) is restrictive, in the sense that functional form assumptions are made, but at least it allows for non-zero correlation between \( c_i \) and the regressors \( x_{it} \).

- The probability that \( y_{it} = 1 \) can now be written as

\[
\Pr (y_{it} = 1|x_{it}, c_i) = \Pr (y_{it} = 1|x_{it}, \bar{x}_i, a_i) = \Phi (x_{it}\beta + \psi + \bar{x}_i\xi + a_i).
\]

You now see that, after having added \( \bar{x}_i \) to the RHS, we arrive at the traditional random effects probit model:

\[
L_i (y_{i1}, \ldots, y_{iT}|x_{i1}, \ldots, x_{iT}; \beta, \sigma_a^2) = \int \prod_{t=1}^{T} [\Phi (x_{it}\beta + \psi + \bar{x}_i\xi + a)]^{y_{it}} \times [1 - \Phi (x_{it}\beta + \psi + \bar{x}_i\xi + a)]^{(1-y_{it})} \phi (a/\sigma_a) da.
\]

- Effectively, we are adding \( \bar{x}_i \) as control variables to allow for some correlation between the random effect \( c_i \) and the regressors.

- If \( x_{it} \) contains **time invariant** variables, then clearly they will be collinear with their mean values for individual \( i \), thus preventing separate identification of \( \beta \)-coefficients on time invariant variables.

- We can easily compute marginal effects at the mean of \( c_i \), since

\[
E (c_i) = \psi + E (\bar{x}_i) \xi
\]

- Notice also that this model nests the simpler and more restrictive traditional random effects probit:
under the (easily testable) null hypothesis that $\xi = 0$, the model reduces to the traditional model discussed earlier.

- EXAMPLE: Exports in Ghana using probit and allowing for unobserved individual effects correlated with mean values of $x$-variables. Appendix Section 3.3

### 3.1.5. Relaxing the normality assumption for the unobserved effect (optional)

The assumption that $c_i$ (or $a_i$) is normally distributed is potentially strong. One alternative is to follow Heckman and Singer (1984) and adopt a **non-parametric** strategy for characterizing the distribution of the random effects. The premise of this approach is that the distribution of $c$ can be approximated by a discrete multinomial distribution with $Q$ points of support:

$$\Pr (c = C_q) = P_q,$$

$0 \leq P_q \leq 1$, $\sum_q P_q = 1$, $q = 1, 2, \ldots, Q$, where the $C_q$, and the $P_q$ are parameters to be estimated.

Hence, the estimated "support points" (the $C_q$) determine possible realizations for the random intercept, and the $P_q$ measure the associated probabilities. The likelihood contribution of individual $i$ is now

$$L_i (y_{i1}, \ldots, y_{iT} | x_{i1}, \ldots, x_{iT}; \beta, \sigma_c^2) = \sum_q P_q \prod_{t=1}^T \left[ \Phi (x_{it} \beta + C_q) \right]^{y_{it}} \left[ 1 - \Phi (x_{it} \beta + C_q) \right]^{(1-y_{it})}.$$  

Compared to the model based on the normal distribution for $c_i$, this model is clearly quite flexible.

In estimating the model, one important issue refers to the number of support points, $Q$. In fact, there are no well-established theoretically based criteria for determining the number of support points in models like this one. Standard practice is to increase $Q$ until there are only marginal improvements in the log likelihood value. Usually, the number of support points is small - certainly below 10 and typically below 5.
Notice that there are many parameters in this model. With 4 points of support, for example, you estimate 3 probabilities (the 4th is a ‘residual’ probability resulting from the constraint that probabilities sum to 1) and 3 support points (one is omitted if - as typically is the case - $x_{it}$ contains a constant). So that’s 6 parameters compared to 1 parameter for the traditional random effects probit based on normality. That is the consequence of attempting to estimate the entire distribution of $c$.

Unfortunately, implementing this model is often difficult:

- Sometimes the estimator will not converge.
- Convergence may well occur at a local maximum.
- Inverting the Hessian in order to get standard errors may not always be possible.

So clearly the additional flexibility comes at a cost.

Allegedly, the Stata program *gllamm* can be used to produce results for this type of estimator.²

### 3.2. Extension: Panel Tobit Models

The treatment of tobit models for panel data is very similar to that for probit models. We state the (non-dynamic) unobserved effects model as

$$y_{it} = \max (0, x_{it}' \beta + c_i + u_{it}) ,$$

$$u_{it} | x_{it}, c_i \sim \text{Normal} \left(0, \sigma^2_u \right).$$

We cannot control for $c_i$ by means of a dummy variable approach (incidental parameters problem), and no tobit model analogous to the "fixed effects" logit exists. We therefore consider the random effects tobit estimator (Note: Bo Honoré has proposed a "fixed effects" tobit that does not impose distributional assumptions. Unfortunately it is hard to implement. Moreover, partial effects cannot be estimated. I therefore do not cover this approach. See Honoré’s web page if you are interested).

²http://www.gllamm.org/
3.2.1. Traditional RE tobit

For the traditional random effects tobit model, the underlying assumptions are the same as those underlying the traditional RE probit. That is,

- $c_i$ and $x_{it}$ are independent
- The $x_{it}$ are strictly exogenous (this will be necessary for it to be possible to write the likelihood of observing a given series of outcomes as the product of individual likelihoods).
- $c_i$ has a normal distribution with zero mean and variance $\sigma_c^2$
- $y_{i1},...,y_{iT}$ are independent conditional on $(x_i, c_i)$, ruling out serial correlation in $y_{it}$, conditional on $(x_i, c_i)$. This assumption can be relaxed.

Under these assumptions, we can proceed in exactly the same way as for the traditional RE probit, once we have changed the log likelihood function from probit to tobit. Hence, the contribution of individual $i$ to the sample likelihood is

$$L_i \left( y_{i1},...,y_{iT}| x_{i1},..., x_{iT}; \beta, \sigma_c^2 \right) =$$

$$\int \prod_{t=1}^{T} \left[ 1 - \Phi \left( \frac{x_{it}\beta + c}{\sigma_u} \right) \right]^{1[y_{it}=0]} \left[ \phi \left( (y_{it} - x_{it}\beta - c)/\sigma_u \right)/\sigma_u \right]^{1[y_{it}=1]} \left( 1/\sigma_c \right) \phi \left( c/\sigma_c \right) dc.$$

This model can be estimated using the `xttobit` command in Stata.

3.2.2. Modelling the random effect as a function of x-variables

The assumption that $c_i$ and $x_{it}$ are independent is unattractive. Just like for the probit model, we can adopt a Mundlak-Chamberlain approach and specify $c_i$ as a function of observables, eg.

$$c_i = \psi + \bar{x}_i \xi + a_i.$$
This means we rewrite the panel tobit as

\[ y_{it} = \max(0, \mathbf{x}_{it}'\mathbf{\beta} + \psi + \bar{\mathbf{x}}_i'\mathbf{\xi} + a_i + u_{it}) , \]

\[ u_{it} | \mathbf{x}_{it}, a_i \sim \text{Normal}(0, \sigma_u^2) \, . \]

From this point, everything is analogous to the probit model (except of course the form of the likelihood function, which will be tobit and not probit) and so there is no need to go over the estimation details again. Bottom line is that we can use the xttobit command and just add individual means of time varying x-variables to the set of regressors. Partial effects of interest evaluated at the mean of \( c_i \) are easy to compute, since

\[ E(c_i) = \psi + E(\bar{x}_i)\mathbf{\xi} . \]

### 3.3. Extension: Heckit with panel data

Model:

\[ y_{it} = \mathbf{x}_{it}'(\mathbf{\beta} + \mathbf{c}_i) + u_{it} , \quad \text{(Primary equation)} \]

where selection is determined by the equation

\[ s_{it} = \begin{cases} 
1 & \text{if } \mathbf{x}_{it}'\mathbf{\gamma} + d_i + v_{it} \geq 0 \\
0 & \text{otherwise} \end{cases} , \quad \text{(Selection equation)} \]

Assumptions regarding unobserved effects and residuals are as for the RE tobit-

- If selection bias arises because \( c_i \) is correlated with \( d_i \), then estimating the main equation using a fixed effects or first differed approach on the selected sample will produce consistent estimates of \( \mathbf{\beta} \).

- However, if \( corr(u_{it}, v_{it}) \neq 0 \), we can address the sample selection problem using a panel Heckit approach. Again, the Mundlak-Chamberlain approach is convenient - that is,
- Write down specifications for $c_i$ and $d_i$ and plug these into the equations above.

- Estimate $T$ different selection probits (i.e. do not use xtprobit here, use pooled probit).
  Compute $T$ inverse Mills ratios.

- Estimate

$$y_{it} = x_{it}\beta + x_i\phi + D_1\rho_1\lambda_1 + \ldots + D_T\rho_T\lambda_T + e_{it},$$

on the selected sample. This yields consistent estimates of $\beta$, provided the model is correctly specified.
1. Derivation of the Inverse Mills Ratio (IMR)

To show

\[ E(z \mid z > c) = \frac{\phi(c)}{1 - \Phi(c)} = \frac{\phi(-c)}{\Phi(-c)} \]

Assume that \( z \) is normally distributed:

\[ G(z) = \Phi(z) \equiv \int_{-\infty}^{z} \phi(z)dz \]

\[ \phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \]

\( G(z) \) is the normal cumulative density function (CDF), \( \phi(z) \) is the standard normal density function.

We now wish to know the \( E(z \mid z > c) \). It is the shaded area in the graph below.

By the characteristics of the normal curve is equal to \([1 - \Phi(c)]\). So the density of \( z \) is given by

\[ \frac{\phi(z)}{[1 - \Phi(c)]}, \quad z > c \]

so
\[ E(z \mid z > c) = \int_c^\infty \frac{z\phi(z)}{[1 - \Phi(c)]} \, dz \]

which can be written using the definitions above as:

\[ E(z \mid z > c) = \frac{1}{(1 - \Phi(c))} \int_c^\infty \frac{z}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \, dz \]

This expression can be written as:

\[ E(z \mid z > c) = \frac{1}{(1 - \Phi(c))} \int_c^\infty \left(\frac{d\phi(z)}{dz}\right) \, dz \]

How do we know that:

\[ \frac{d\phi(z)}{dz} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right), -z \]

\[ \int_c^\infty \left(\frac{d\phi(z)}{dz}\right) \, dz = \int_c^\infty -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \, dz = 0 + \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{c^2}{2}\right) = \phi(c) \]

So:

Let’s evaluate \( \int_c^\infty \frac{z}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \, dz = \)

This can be written as

\[ -\frac{1}{[1 - \Phi(c)]} \int_c^\infty d\Phi(z) = \frac{\phi(c)}{[1 - \Phi(c)]} \]

Recall that for the normal distribution \( \phi(c) = \phi(-c) \) and \( 1 - \Phi(c) = \Phi(-c) \)

From which it follows that

\[ E(z \mid z > c) = \int_c^\infty \frac{z\phi(z)}{[1 - \Phi(c)]} \, dz = \frac{\phi(-c)}{\Phi(-c)} \]

It is this last expression which is the inverse Mills ratio.
Figure 1: The Inverse Mills Ratio
The economic model underlying the graph is
\[ \ln w = \text{cons} + 0.1 \text{educ} + m, \]
where \( w \) is wage, \( \text{educ} \) is education and \( m \) is unobserved motivation.
2. Two empirical illustrations of the Heckit model

2.1 Earnings regressions for wage-employed men aged 16-30 in Pakistan


Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>lw</td>
<td>4853</td>
<td>10.10</td>
<td>.701</td>
<td>4.78</td>
<td>12.87</td>
</tr>
<tr>
<td>educ</td>
<td>10018</td>
<td>5.89</td>
<td>4.72</td>
<td>0</td>
<td>19</td>
</tr>
<tr>
<td>age</td>
<td>10018</td>
<td>22.84</td>
<td>4.34</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>married</td>
<td>10018</td>
<td>.38</td>
<td>.48</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>kidsund12</td>
<td>10018</td>
<td>2.57</td>
<td>2.63</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

| eldove65 | 10018| .99     | .46       | 0     | 3     |

i) OLS

. reg lw educ age married if sex==1 & age<=30

Source | SS   | df | MS  | Number of obs = 4853
-------|-----|----|-----|----------------------
Model  | 565.75| 3  | 188.58 | F( 3, 4849) = 501.00
Residual | 1825.23| 4849 | .3764 | R-squared = 0.2366
Total  | 2390.98| 4852 | .4928 | Adj R-squared = 0.2361

Root MSE = .6135

| lw | Coef. | Std. Err. | t   | P>|t|  | [95% Conf. Interval] |
|----|-------|-----------|-----|-----|---------------------|
| educ | .0370919 | .0018438 | 20.12 | 0.000 | .0334772 - .0407066 |
| age  | .0508328 | .0025309 | 20.09 | 0.000 | .0458711 - .0557944 |
| married | .1569765 | .0216604 | 7.25  | 0.000 | .1145123 - .1994407 |
| _cons | 8.614617 | .0544189 | 158.30 | 0.000 | 8.507931 - 8.721303 |
ii) Heckit

. heckman lw educ age married if sex==1 & age<=30, select(age educ kidsund12 eldove65 married) twostep

Heckman selection model -- two-step estimates
(regression model with sample selection)

|                     | Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|---------------------|-------|-----------|-------|------|----------------------|
| lw                  |       |           |       |      |                      |
| educ                | 0.036956 | 0.0018689 | 19.77 | 0.000 | 0.0332929 0.040619     |
| age                 | 0.0494845 | 0.0038901 | 12.72 | 0.000 | 0.0418601 0.0571088   |
| married             | 0.154299 | 0.0224542 | 6.87  | 0.000 | 0.1102895 0.1983084   |
| _cons               | 8.68906  | 0.1718886 | 50.55 | 0.000 | 8.352165 9.025956     |
| select              |       |           |       |      |                      |
| age                 | 0.0406764 | 0.0035879 | 11.34 | 0.000 | 0.0336443 0.0477085   |
| educ                | 0.003192  | 0.0027291 | 1.17  | 0.242 | -0.0021569 0.008541   |
| kidsund12           | -0.0487458 | 0.0050142 | -9.72 | 0.000 | -0.0585734 -0.0389181  |
| eldove65            | -0.0580974 | 0.0276959 | -2.10 | 0.036 | -0.1123805 -0.0038144  |
| married             | 0.1366124  | 0.0325453 | 4.20  | 0.000 | 0.0728248 0.2003999   |
| _cons               | -0.9026498 | 0.0775054 | -11.65| 0.000 | -1.054558 0.7507419   |
| mills               |       |           |       |      |                      |
| lambda              | -0.0509581 | 0.1115998 | -0.46 | 0.648 | -0.2696897 0.1677734  |
| rho                 | -0.08291  |           |       |      |                      |
| sigma               | 0.61459573 |           |       |      |                      |
| lambda              | -0.0509581 | 0.1115998 | -0.46 | 0.648 | -0.2696897 0.1677734  |
2.2 Earnings regressions for females in the US

This section uses the MROZ dataset.¹ This dataset contains information on 753 women. We observe the wage offer for only 428 women, hence the sample is truncated.

use C:\teaching_gbg07\applied_econ07\MROZ.dta

1. OLS on selected sample

reg lwage educ exper expersq

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 428</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>35.0223023</td>
<td>3</td>
<td>11.6741008</td>
<td>F(  3,   424) = 26.29</td>
</tr>
<tr>
<td>Residual</td>
<td>188.305149</td>
<td>424</td>
<td>.444115917</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>223.327451</td>
<td>427</td>
<td>.523015108</td>
<td>Root MSE = .66642</td>
</tr>
</tbody>
</table>

| lwage | Coef.  | Std. Err. | t    | P>|t| |   [95% Conf. Interval] |
|-------|--------|-----------|------|------|-----------------------|
| educ  | .1074896 | .0141465  | 7.60 | 0.000 | .0796837 - .1352956   |
| exper | .0415665 | .0131752  | 3.15 | 0.002 | .0156697 - .0674633   |
| expersq | -.0008112 | .0003932 | -2.06 | 0.040 | -.0015841 - .0000382 |
| _cons | -.5220407 | .1986321  | -2.63 | 0.009 | -.9124668 - .1316145 |

2. Two-step Heckit

```
. heckman lwage educ exper expersq, select(nwifeinc educ exper expersq age kidslt6 kidsge6) twostep

Heckman selection model -- two-step estimates
(regression model with sample selection)
Number of obs      =       753
Censored obs       =       325
Uncensored obs     =       428
Wald chi2(6)       =    180.10
Prob > chi2        =    0.0000

------------------------------------------------------------------------------
|      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lwage        |
    educ |   .1090655    .015523     7.03   0.000     .0786411      .13949
    exper |   .0438873   .0162611     2.70   0.007     .0120163    .0757584
   expersq |  -.0008591   .0004389    -1.96   0.050    -.0017194    1.15e-06
     _cons |  -.5781033   .3050062    -1.90   0.058    -.1175904    .0196979
-------------+----------------------------------------------------------------
  select      |
    nwifeinc |  -.0120237   .0048398    -2.48   0.013    -.0215096   -.0025378
    educ |   .1309047   .0252542     5.18   0.000     .0814074     .180402
    exper |   .1233476   .0187164     6.59   0.000     .0866641    .1600311
   expersq |  -.0018871      .0006    -3.15   0.002     -.003063   -.0007111
     age |  -.0528527   .0084772    -6.23   0.000    -.0694678   -.0362376
  kidslt6 |  -.8683285   .1185223    -7.33   0.000    -1.100628    -.636029
   kidsge6 |   .036005   .0434768     0.83   0.408     -.049208    .1212179
     _cons |   .2700768    .508593     0.53   0.595    -.7267472    1.266901
-------------+----------------------------------------------------------------
    mills     |
    lambda |   .0322619   .1336246     0.24   0.809     -.2296376    .2941613
-------------+----------------------------------------------------------------
    rho     |   0.04861
    sigma |   .66362876
    lambda |   .03226186   .1336246
------------------------------------------------------------------------------
```
### 3. Simultaneous estimation of selection model

```
. heckman lwage educ exper expersq, select(nwifeinc educ exper expersq age kidslt6 kidsge6)
```

| Coef.   | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|---------|-----------|------|------|----------------------|
| lwage   |           |      |      |                      |
| educ    | 0.1083502 | 0.0148607 | 7.29 | 0.000 | 0.0792238 - 0.1374767 |
| exper   | 0.0428369 | 0.0148785 | 2.88 | 0.004 | 0.0136755 - 0.0719983 |
| expersq | -0.0008374 | 0.0004175 | -2.01 | 0.045 | -0.0016556 - 0.0000192 |
| _cons   | -0.5526974 | 0.2603784 | -2.12 | 0.034 | -1.06303 - 0.0423652 |

| select  |           |      |      |                      |
| nwifeinc | -0.012132 | 0.0048767 | -2.49 | 0.013 | -0.0216903 - 0.002574 |
| educ    | 0.1313415 | 0.0253823 | 5.17 | 0.000 | 0.0815931 - 0.1810899 |
| exper   | 0.1232818 | 0.0187242 | 6.58 | 0.000 | 0.0865831 - 0.1599806 |
| expersq | -0.0018863 | 0.0006004 | -3.14 | 0.002 | -0.003063 - 0.0007095 |
| age     | -0.0528287 | 0.0084792 | -6.23 | 0.000 | -0.0694476 - 0.0362098 |
| kidslt6 | -0.8673988 | 0.1186509 | -7.31 | 0.000 | -1.09995 - 0.6348472 |
| kidsge6 | 0.0358723 | 0.0434753 | 0.83 | 0.409 | -0.0493777 - 0.1210824 |
| _cons   | 0.2664491 | 0.5089578 | 0.52 | 0.601 | -0.7310898 - 1.263988 |

/athrho | 0.026614 | 0.147182 | 0.18 | 0.857 | -0.2618573 - 0.3150854 |

/Lnsigma | -0.4103809 | 0.0342291 | -11.99 | 0.000 | -0.4774687 - 0.3432931 |

| rhol    | 0.0266078 | 0.1470778 |                      |
| sigmal  | 0.6633975 | 0.0227075 |                      |
| lambdal | 0.0176515 | 0.0976057 |                      |

LR test of indep. eqns. (rho = 0): chi2(1) = 0.03 Prob > chi2 = 0.8577
4. Selection model with endogenous education

```
. probit inlf nwifeinc exper expersq age kidslt6 kidsge6 motheduc fatheduc huseduc

Iteration 0:   log likelihood = -514.8732
Iteration 1:   log likelihood = -414.44513
Iteration 2:   log likelihood = -411.33354
Iteration 3:   log likelihood = -411.32238
Iteration 4:   log likelihood = -411.32238

Probit regression                               Number of obs   =        753
LR chi2(9)      =     207.10
Prob > chi2     =     0.0000
Log likelihood = -411.32238                       Pseudo R2       =     0.2011

------------------------------------------------------------------------------
inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
nwifeinc |  -.0074294   .0048787    -1.52   0.128    -.0169915    .0021327
exper   |   .1285092   .0185226     6.94   0.000     .0922056    .1648129
expersq  |  -.0019474   .0005955    -3.27   0.001    -.0031146   -.0007803
age     |  -.0527657   .0085423    -6.18   0.000    -.0695082   -.0360231
kidslt6 |  -.8149255   .1160833    -7.02   0.000    -1.042445   -.5874063
kidsge6 |   .0241511   .0432253     0.56   0.576     -.060569    .1088712
motheduc |   .0295321   .0185718     1.59   0.112     -.006868    .0659322
fatheduc |   .0133487   .0178491     0.75   0.455    -.0216349    .0483324
huseduc  |   .0161391    .019595     0.82   0.410    -.0222664    .0545446
_cons    |   1.146672   .4932706     2.32   0.020     .1798798    2.113465
------------------------------------------------------------------------------
```

. predict zg, xb
. ge imr=normalden(zg)/normal(zg)

. ivreg2 lwage exper expersq imr (educ = nwifeinc exper expersq age kidslt6 kidsge6 motheduc fatheduc huseduc)

---

10
Warning - duplicate variables detected
Duplicates: exper expersq

IV (2SLS) estimation

Estimates efficient for homoskedasticity only
Statistics consistent for homoskedasticity only

|                | Coef. | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|----------------|-------|-----------|-------|-------|----------------------|
| lwage          |       |           |       |       |                      |
| educ           | 0.0877632 | 0.0212981 | 4.12  | 0.000 | 0.0460196 - 0.1295067 |
| exper          | 0.0457425 | 0.0164923 | 2.77  | 0.006 | 0.0134182 - 0.0780664 |
| expersq        | -0.0009128 | 0.0004441 | -2.06 | 0.040 | -0.0017833 - 0.0000423 |
| imr            | 0.0404355 | 0.1326462 | 0.30  | 0.760 | -0.2195463 - 0.3004173 |
| _cons          | -0.3249135 | 0.3315012 | -0.98 | 0.327 | -0.974644 - 0.324817 |

Underidentification test (Anderson canon. corr. LM statistic): 188.702
Chi-sq(7) P-val = 0.0000

Weak identification test (Cragg-Donald Wald F statistic): 46.976
Stock-Yogo weak ID test critical values: 5% maximal IV relative bias 19.86
10% maximal IV relative bias 11.29
20% maximal IV relative bias 6.73
30% maximal IV relative bias 5.07
10% maximal IV size 31.50
15% maximal IV size 17.38
20% maximal IV size 12.48
25% maximal IV size 9.93


Sargan statistic (overidentification test of all instruments): 6.961
Chi-sq(6) P-val = 0.3245

Instrumented: educ
Included instruments: exper expersq imr
Excluded instruments: nwifeinc age kidslt6 kidsge6 motheduc fatheduc huseduc
Duplicates: exper expersq
3. Panel probit estimation

In the following examples we consider a model of the binary decision to export, using firm-level data from Ghana’s manufacturing sector.\(^2\)

3.1. Traditional random effects probit: Individual effects uncorrelated with regressors

> xi: xtprobit exports lyl lkl le anyfor i.year i.town i.industry;

Fitting comparison model:

Iteration 0:   log likelihood = -408.96484
(…)
Iteration 4:   log likelihood = -253.77448

Fitting full model:

rho = 0.0  log likelihood = -253.77448
(…)
rho = 0.6  log likelihood = -219.85297

Iteration 0:   log likelihood = -218.83642
(…)
Iteration 6:   log likelihood = -199.16376

Random-effects probit regression

<table>
<thead>
<tr>
<th>Group variable: firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs = 802</td>
</tr>
<tr>
<td>Number of groups = 209</td>
</tr>
</tbody>
</table>

Random effects \( u_i \) ~ Gaussian

| Obs per group: min = 1 |
| avg = 3.8 |
| max = 5 |

Wald chi2(16) = 44.64

Log likelihood = -199.16376

Prob > chi2 = 0.0002

|                  | Coef. | Std. Err. | z     | P>|z|    | [95% Conf. Interval] |
|------------------|-------|-----------|-------|--------|----------------------|
| exports | lyl | .1323961 | .1215347 | 1.09 | 0.276 | -.1058075 .3705997 |
| exports | lkl | .1758152 | .1345334 | 1.31 | 0.191 | -.0878654 .4394958 |
| exports | le  | .6649512 | .1785642 | 3.72 | 0.000 | .3149718 1.014931 |
| exports | anyfor | -.3167296 | .5233455 | -0.61 | 0.545 | -.8021741 .1687184 |
| exports | _Iyear_1996 | -.070118 | .3138138 | -0.22 | 0.823 | -.6851817 .5449458 |
| exports | _Iyear_1997 | -.3726546 | .3014128 | -1.24 | 0.216 | -.9634129 .2181037 |
| exports | _Iyear_1998 | .1571592 | .295838 | 0.53 | 0.595 | -.4226727 .7369911 |
| exports | _Iyear_1999 | -.035217 | .7820001 | -0.12 | 0.896 | -.1693562 .1089218 |
| exports | _Itown_2 | .1854388 | .7820001 | 0.24 | 0.813 | -.1347253 .1718131 |
| exports | _Itown_3 | -.2783668 | .5239421 | -0.53 | 0.595 | -.1305274 .7485408 |
| exports | _Itown_4 | .615539 | 1.241932 | 0.50 | 0.620 | -.8186024 3.04968 |
| exports | _Iindustry_2 | 4.550127 | .9358409 | 4.2 | 0.000 | 2.715913 6.348432 |
| exports | _Iindustry_3 | .7986741 | .9358409 | 0.41 | 0.679 | -.1029066 2.698414 |
| exports | _Iindustry_4 | .083837 | .7961364 | 0.11 | 0.916 | -.1476562 1.644236 |
| exports | _Iindustry_5 | .4340664 | .6577467 | 0.66 | 0.509 | -.8550933 1.723226 |
| exports | _Iindustry_6 | .5219096 | .5584191 | 0.93 | 0.350 | -.5725717 1.616391 |
| exports | _cons | -7.341894 | 1.625434 | -4.52 | 0.000 | -10.52769 -4.156102 |

\[
\text{\textbackslash{insig2u}} \mid 1.197964 \quad .336149 \quad .5391236 \quad 1.856803
\]
\[
sigma_u \mid 1.820264 \quad .30594 \quad 1.309391 \quad 2.530462
\]
\[
rho \mid .7681623 \quad .0598644 \quad .6316085 \quad .8649239
\]

Likelihood-ratio test of \( \rho=0 \): chibar2(01) = 109.22 Prob >= chibar2 = 0.000

### 3.2. Panel fixed effects logit: Individual effects freely correlated with regressors

```
. xi: xtlogit exports lyl lkl le anyfor i.year i.town i.industry, fe;
```

Conditional fixed-effects logistic regression
Number of obs = 142
Number of groups = 32
Obs per group: min = 3
avg = 4.4
max = 5
LR chi2(7) = 17.71
Prob > chi2 = 0.0134

```
|          |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval] |
|-----------|-----------------|---------|-------|-----------------|
| lyl       | -.1775995       .3069105  -0.58  0.563    -.779133    .4239341 |
| lkl       | 12.89616        4.428396   2.91  0.004     4.216661    21.57566 |
| le        | 12.89509        4.333415   2.98  0.003     4.401752    21.38843 |
| _Iyear_1996 | .9050252     .692148      1.31  0.191    -.4515599    2.26161 |
| _Iyear_1997 | .2076181     .6228052     0.33  0.739    -1.013058    1.428294 |
| _Iyear_1998 | 1.205249      .6522533    1.85  0.065     -.0731435    2.483642 |
| _Iyear_1999 | .4657296      .603257     0.77  0.440    -.7166324    1.648092 |
```

Drop lyl lkl to see if we can get something more meaningful.

```
. xi: xtlogit exports  le i.year , fe;
```

Conditional fixed-effects logistic regression
Number of obs = 157
Number of groups = 34
Obs per group: min = 3
avg = 4.6
max = 5
LR chi2(5) = 6.40
Prob > chi2 = 0.2690

```
|          |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval] |
|-----------|-----------------|---------|-------|-----------------|
| le        | 1.022553        .5995129   1.71  0.088    -.1524704    2.197577 |
| _Iyear_1996 | -.0550044     .547602   -0.10  0.920    -1.128285    1.018276 |
| _Iyear_1997 | -.5514746     .5123415  -1.08  0.282    -1.555645    .4526964 |
| _Iyear_1998 | .2639011       .4787566   0.55  0.581    -.6744445    1.202247 |
| _Iyear_1999 | .0541304       .4871164   0.11  0.912    -.9006001    1.008861 |
```
### 3.3. Panel random effects probit: Individual effects correlated with mean values of regressors

\[ egen mlyl=mean(lyl), by(firm); \]
\[ (344 \text{ missing values generated}) \]
\[ egen mlkl=mean(lkl), by(firm); \]
\[ (470 \text{ missing values generated}) \]
\[ egen mle =mean(le), by(firm); \]
\[ (339 \text{ missing values generated}) \]
\[ xi: xtprobit exports lyl lkl le anyfor mlyl mlkl mle i.year i.town \]

Random-effects probit regression
---
Number of obs = 802
Number of groups = 209

Random effects u_i ~ Gaussian
Obs per group: min = 1
avg = 3.8
max = 5

Wald chi2(19) = 38.53
Prob > chi2 = 0.0051

| exports | Coef.  | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|---------|--------|-----------|-------|----|---|----------------------|
| lyl     | 0.001653 | 0.15102 | 0.01  | 0.991 | -0.2943408 | 0.2976467 |
| lkl     | 2.659183 | 1.287121 | 2.07  | 0.039 | 0.1364717 | 5.181895 |
| le      | 2.919679 | 1.306894 | 2.23  | 0.025 | 0.3582138 | 5.481145 |
| anyfor  | -0.3607094 | 0.5659661 | -0.64 | 0.524 | -1.469983 | 0.7485639 |
| mlyl    | 0.4115834 | 0.2877433 | 1.43  | 0.153 | -0.1523831 | 0.9755499 |
| mlkl    | -2.546631 | 1.289167 | -1.98 | 0.048 | -5.073353 | -0.0199097 |
| mle     | -2.468686 | 1.294889 | -1.74 | 0.083 | -4.748431 | 0.2910394 |
| _Iyear_1996 | 0.2118903 | 0.3523389 | 0.60  | 0.548 | -0.4786813 | 0.9024619 |
| _Iyear_1997 | -1.186531 | 0.3250747 | -0.57 | 0.566 | -0.8236658 | 0.4506037 |
| _Iyear_1998 | 0.3610692 | 0.3237451 | 1.12  | 0.265 | -0.2734617 | 0.9956002 |
| _Iyear_1999 | 0.0897934 | 0.3237462 | 0.29  | 0.775 | -0.5265664 | 0.7061532 |
| _Itown_2 | 2.240639 | 0.8509283 | 2.62  | 0.009 | 0.6523256 | 1.838243 |
| _Itown_3 | -0.3241008 | 0.5582742 | -0.58 | 0.562 | -1.418298 | 0.7700964 |
| _Itown_4 | 1.029035 | 1.343127 | 0.77  | 0.444 | -1.603445 | 3.661515 |
| _Iindustry_2 | 5.275066 | 1.143112 | 4.61  | 0.000 | 3.034608 | 7.515524 |
| _Iindustry_3 | 0.8018585 | 1.046514 | 0.77  | 0.444 | -1.249271 | 2.852988 |
| _Iindustry_4 | 0.5816888 | 0.7316163 | 0.80  | 0.427 | -0.8522528 | 2.015631 |
| _Iindustry_5 | 0.5398758 | 0.604692 | 0.90  | 0.369 | -0.637023 | 1.716774 |
| _Iindustry_6 | -0.3241008 | 0.5582742 | -0.58 | 0.562 | -1.418298 | 0.7700964 |
| _cons   | -9.377469 | 2.3808797 | -3.94 | 0.000 | -14.04391 | -4.711032 |
| /lnsig2u | 1.340067 | 0.3496672 |       |    | 0.654732 | 2.025403 |
| sigma_u | 1.954303 | 0.3416779 |       |    | 1.387309 | 2.753028 |
| rho     | 0.792501 | 0.0575004 |       |    | 0.658076 | 0.8834385 |

Likelihood-ratio test of rho=0: chibar2(01) = 113.10
Prob >= chibar2 = 0.000