Pursuing the wrong options? Adjustment costs and the relationship between uncertainty and capital accumulation: Online Appendix

Stephen R. Bond[1], Måns Söderbom[2] and Guiying Wu[3]
January 2011

This appendix describes the numerical procedure used to generate the simulated investment data analyzed in Bond, Söderbom and Wu (2011). It also contains additional results.

The model

The firm's objective is to maximize the value of the firm:

$$V(X_t, K_t) = \max_{I_t}\{\Pi(X_t, K_t; I_t) + \frac{1}{1+r} E_t \left[V(X_{t+1}, K_{t+1})\right]\}$$

where

$$\Pi(X_t, K_t; I_t) = \frac{h}{1-\gamma} X_t^{\gamma} \left(K_t + I_t\right)^{1-\gamma} - I_t - G\left(I_t, K_t\right),$$

$$K_{t+1} = I_t + K_t,$$

and

$$\begin{aligned} \ln X_t &= \ln X_{t-1} + \tilde{\mu} + \varepsilon_t \\ \varepsilon_t &\sim \text{iid } N\left(0, \sigma^2\right) \\ \ln X_0 &= 0 \\ \tilde{\mu} &= \mu - 0.5\sigma^2, \end{aligned}$$

(see the main paper for definitions).

Exploiting linear homogeneity of the value function

Since the value function is homogeneous of degree 1 in (X, K), we can eliminate K_t from the state space by normalizing by K_t. Defining $\pi_t = \Pi_t/K_t = \frac{h}{1-\gamma}\left(x_t\right)^{\gamma}\left(1 + I_t/K_t\right)^{1-\gamma}$, $x_t = X_t/K_t$, $v_t = V_t/K_t$, we thus write the value of the firm normalized by K_t as

$$v(x_t) = \max_{I_t/K_t}\{\pi\left(x_t; I_t/K_t\right) + \frac{I_t/K_t + 1}{1+r} E_t v\left(x_{t+1}\right)\}. \quad (1)$$

The same approach has been used by Bloom (2009).

[1] Nuffield College, Department of Economics and Centre for Business Taxation, University of Oxford, UK, and Institute for Fiscal Studies. E-mail: steve.bond@economics.ox.ac.uk
[2] Department of Economics, University of Gothenburg, Sweden. E-mail: mans.soderbom@economics.gu.se. Corresponding author.
[3] Division of Economics, Nanyang Technological University, Singapore. E-mail: guiying.wu@ntu.edu.sg

Solving the Bellman equation using value function iteration

We solve the firm's optimization problem using value function iteration. The principles of our algorithm are as follows.

1. Start with a guess for the true value function $v(x_t)$. Call this guess v^1. Use it on the right-hand side of the Bellman equation (1), and compute $E_t[v(x_{t+1})]$ (more on how to compute this expectation below). Find the values of I_t/K_t that maximize firm value given the state x_t.

2. Update the true value function $v(x_t)$ using the solution obtained in the previous step (i.e. this is the value v associated with optimal I_t/K_t). Call this updated guess v^2. Check if $v^2 = v^1$; if true, we have converged to the true function and so iteration can stop; if not set $j = 2$ and go to step 3.

3. Set $j = j + 1$. Use v^{j-1} on the right-hand side of the Bellman equation to compute $E_t[v(x_{t+1})]$, and calculate the optimal choice rule I_t/K_t. Update the value function, v^j. Check if $v^j = v^{j-1}$; If true, there is convergence and so iteration stops; if not, repeat step 3.

To implement this method we need to discretize the state and control space, and we need a way of calculating the expected value of the firm in $t + 1$. We discretize the distribution of the variables x_t (state) and x_{t+1} (control) by constructing a finite set of permissible values $(x_1, x_2, ..., x_M)$ where x_1 is the lowest permissible value, x_M the highest, and M is the number of permissible values. In the simulations we vary x_1 and x_M depending on the size of the adjustment cost parameter b_q and the level of uncertainty σ The limits are set sufficiently wide not to bind, whilst trying to avoid too extreme values as this would result in a coarse grid. We use $M = 300$ throughout. Increasing M further does not affect the results. The expectation of the value function $E_t[v(x_{t+1})]$ is computed by summing over the M possible realizations of the value function associated with the possible realizations of x_{t+1} that follow from the stochastic process using weights implied by the normal distribution.

The simulated data

Once the value iteration has converged, we recover the policy functions for investment. Based on these we simulate J panel data sets on N firms over T time periods. We use $J = 8$, $T = 100$ and different random numbers throughout. We use larger values of N for higher values of σ so as to keep standard errors of the simulated means reasonably low. We compute the mean and the standard error of $\kappa(T = 100)$ using the following formulae:

$$\frac{1}{J}\sum_{j=1}^{J}\kappa(T=100)_j \quad \text{(mean)},$$

$$\frac{1}{J}\sqrt{\sum_{j=1}^{J}\left\{se\left[\kappa(T=100)\right]_j\right\}^2} \quad \text{(standard error)},$$

where

$$\kappa\left(T=100\right)_j = \left[\left[h\left(1+r\right)/r\right]^{(1/\gamma)} e^{\mu\cdot 100}\right]^{-1} \left\{\frac{1}{n}\sum_n K\left(T=100\right)\right\}_j ,$$

$$se\left[\kappa\left(T=100\right)\right]_j = \left[\left[h\left(1+r\right)/r\right]^{(1/\gamma)} e^{\mu\cdot 100}\right]^{-1} \sqrt{\left(\frac{Var(K\left(T=100\right))}{N}\right)_j}.$$

Additional analysis

Simulating the Abel-Eberly model. We compute numerical simulations of κ based on the Abel-Eberly model (which features complete irreversibility and no quadratic adjustment costs) and compare these to the analytical values, which are given by

$$\kappa\left(\infty\right) = \left(1 - \frac{\gamma}{\alpha_N}\right)^{-1/\gamma} \times \left(1 + \sigma^2/2\mu\right), \tag{2}$$

where

$$\alpha_N = \sigma^2 - 2\mu - \frac{\sqrt{((2\mu - \sigma^2) + 8r\sigma^2)}}{2\sigma^2},$$

is the negative root of equation (5b) in Aberly and Eberly (1999). The results are shown in Table A1 and Figure A1. The simulated values of κ track the analytical values closely, suggesting there are no major errors in our computer code. For large values of σ, we need a very large number of firms to obtain reasonable precision in the simulated means. This is much less of an issue for the quadratic costs model, since firms in that model can adjust capital downwards.

Additional results. The full set of results underlying the summary statistics shown in Table 1 in the main paper are shown in Table A2. We show values of κ for the real options model based on (2), and simulated values of κ for the quadratic adjustment costs model. For the latter we also show standard errors.

Abel and Eberly (1999) discuss the effects of changing other model parameters on long run capital accumulation. They prove that r has a positive effect and that μ and η have negative effects. Table A3 shows the results of regressing the values of $\kappa\left(T\right)$ obtained for different values of the parameters σ, η, μ, r on these parameter values. The first column shows the results for the real options model. The results are consistent with the theoretical results documented in Abel and Eberly (1999), Section 5. The effects of uncertainty is small. An increase in the parameter η has a small negative effect on $\kappa\left(T\right)$. An increase in the discount rate brings expected long run capital closer to the level that would prevail under reversible investment.[4] Demand growth has the opposite effect.

[4] Note that this does not imply that a higher discount rate has a positive effect on the level of capital. An increase in r reduces capital accumulation under reversibility as well as irreversibility. The result simply indicates that this negative effect is weaker under irreversibility, thus increasing $\kappa\left(T\right)$.

Columns (2) and (3) show the results for the models with quadratic costs. We find a significant negative effect of uncertainty on $\kappa(T)$, which is stronger for $b_q = 3.0$ than for $b_q = 0.5$. This suggests that the effect of higher uncertainty on long run capital accumulation is more adverse if adjustment costs are high. An increase in η tends to reduce $\kappa(T)$, i.e. increase the gap between long run expected capital under quadratic costs and the level without adjustment costs. An increase in the discount rate r has the opposite effect, similar to the real options model.

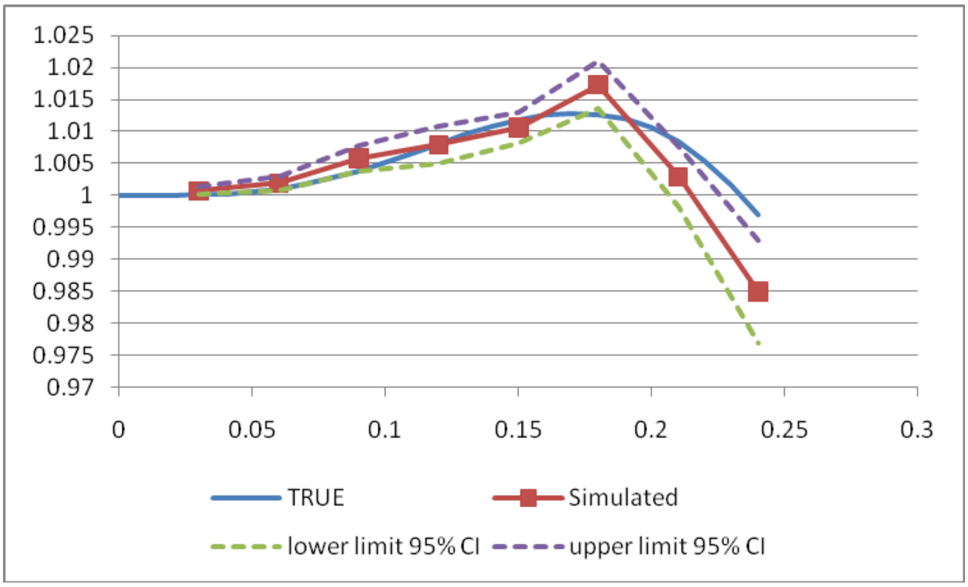
References

[1] Abel, Andrew B. and Eberly, Janice C. (1999), "The Effects of Irreversibility and Uncertainty on Capital Accumulation", *Journal of Monetary Economics*, 44, 339-377.

[2] Bloom, Nicholas (2009), "The impact of uncertainty shocks", *Econometrica* 77: 623-685.

[3] Bond, Steve, Söderbom, Måns and Wu, Guiying (2011). "Pursuing the Wrong Options? Adjustment Costs and the Relationship between Uncertainty and Capital Accumulation", forthcoming, *Economics Letters.*

Figure A1: Comparing Simulated and True Values of κ in the Abel-Eberly (1999) Model



Note: See Table A1 for further details.

Table A1: Comparing Simulated and True Values of κ in the Abel-Eberly (1999) Model

σ	True κ	Simulated κ	Std error	N	J	Grid points
0.03	1.0001	1.0007	0.0003	12500	100	500
0.06	1.001	1.0019	0.0006	12500	100	500
0.09	1.0038	1.0058	0.001	12500	100	500
0.12	1.008	1.0079	0.0015	12500	100	500
0.15	1.0118	1.0106	0.0012	50000	100	500
0.18	1.0127	1.0173	0.0019	50000	100	500
0.21	1.0083	1.0029	0.0024	100000	100	1000
0.24	0.9969	0.9849	0.0041	100000	100	1000

Table A2: The Long Run Effect of Uncertainty on Capital Accumulation: Full Set of Results

				Real Options	Quadratic costs: $b_q = 0.5$		Quadratic costs: $b_q = 3.0$	
mu	r	eta	sigma	κ	κ	Std error	κ	Std error
0.02	0.05	5	0.01	1	1.0004	0.0007	0.9981	0.0006
0.02	0.05	5	0.06	1.0067	0.9942	0.002	0.9828	0.0018
0.02	0.05	5	0.12	1.0511	0.9805	0.0055	0.9361	0.0046
0.02	0.05	5	0.18	1.1207	0.959	0.0089	0.8168	0.0038
0.02	0.05	5	0.24	.	0.9264	0.0141	0.6631	0.0044
0.029	0.05	5	0.01	1	1.0005	0.0007	0.999	0.0006
0.029	0.05	5	0.06	1.0013	0.9928	0.002	0.9825	0.0018
0.029	0.05	5	0.12	1.012	0.9776	0.0052	0.9372	0.0045
0.029	0.05	5	0.18	1.0284	0.9506	0.0071	0.8264	0.0041
0.029	0.05	5	0.24	1.0352	0.9047	0.0124	0.6669	0.0049
0.038	0.05	5	0.01	1	1.0009	0.0007	0.9988	0.0006
0.038	0.05	5	0.06	1	0.9904	0.002	0.9806	0.0018
0.038	0.05	5	0.12	0.9995	0.9725	0.0051	0.9365	0.0043
0.038	0.05	5	0.18	0.9933	0.9475	0.0061	0.8247	0.004
0.038	0.05	5	0.24	0.9739	0.9008	0.0119	0.6777	0.0053
0.047	0.05	5	0.01	1	1.0039	0.0007	0.9996	0.0007
0.047	0.05	5	0.06	0.9997	0.995	0.002	0.9824	0.0018
0.047	0.05	5	0.12	0.9954	0.9732	0.0052	0.9297	0.0044
0.047	0.05	5	0.18	0.9798	0.9453	0.0065	0.8303	0.0045
0.047	0.05	5	0.24	0.947	0.9035	0.0217	0.6783	0.0052
0.02	0.05	10	0.01	1	1.0007	0.0007	0.9968	0.0006
0.02	0.05	10	0.06	1.0061	0.9921	0.002	0.9726	0.0017
0.02	0.05	10	0.12	1.0449	0.9572	0.0049	0.9058	0.0039
0.02	0.05	10	0.18	1.0982	0.925	0.007	0.7115	0.0025
0.02	0.05	10	0.24	.	0.8432	0.0088	0.488	0.002
0.029	0.05	10	0.01	1	1.0001	0.0007	0.9965	0.0006
0.029	0.05	10	0.06	1.001	0.9888	0.002	0.9743	0.0017
0.029	0.05	10	0.12	1.008	0.9602	0.0049	0.9038	0.0039
0.029	0.05	10	0.18	1.0127	0.9136	0.0061	0.7125	0.0026
0.029	0.05	10	0.24	0.9969	0.8674	0.0106	0.499	0.0021
0.038	0.05	10	0.01	1	1.0015	0.0007	0.9947	0.0006
0.038	0.05	10	0.06	0.9998	0.9892	0.0019	0.9719	0.0017
0.038	0.05	10	0.12	0.9968	0.9711	0.0051	0.8995	0.0039
0.038	0.05	10	0.18	0.9819	0.9096	0.0066	0.7106	0.0027
0.038	0.05	10	0.24	0.9445	0.8454	0.0113	0.4989	0.0022
0.047	0.05	10	0.01	1	1.0014	0.0007	0.9967	0.0006
0.047	0.05	10	0.06	0.9996	0.9934	0.002	0.975	0.0017
0.047	0.05	10	0.12	0.9935	0.96	0.005	0.9105	0.0041
0.047	0.05	10	0.18	0.9711	0.9145	0.006	0.7219	0.0029
0.047	0.05	10	0.24	0.9236	0.8542	0.0109	0.497	0.0023
0.02	0.05	20	0.01	1	1.001	0.0007	0.9918	0.0005

0.02	0.05	20	0.06	1.0057	0.9856	0.0019	0.9587	0.0014
0.02	0.05	20	0.12	1.0408	0.951	0.0048	0.8582	0.003
0.02	0.05	20	0.18	1.0827	0.8632	0.0048	0.587	0.0014
0.02	0.05	20	0.24	.	0.7473	0.0069	0.3483	0.0008
0.029	0.05	20	0.01	1	1.0016	0.0007	0.9889	0.0006
0.029	0.05	20	0.06	1.0008	0.9868	0.0019	0.9512	0.0015
0.029	0.05	20	0.12	1.0054	0.9542	0.0048	0.8456	0.003
0.029	0.05	20	0.18	1.0021	0.8738	0.0053	0.5754	0.0014
0.029	0.05	20	0.24	0.9701	0.7597	0.0089	0.3449	0.0009
0.038	0.05	20	0.01	1	1.0007	0.0007	0.9888	0.0006
0.038	0.05	20	0.06	0.9997	0.9869	0.0019	0.9571	0.0015
0.038	0.05	20	0.12	0.995	0.949	0.0048	0.8521	0.0032
0.038	0.05	20	0.18	0.9741	0.8727	0.0052	0.575	0.0015
0.038	0.05	20	0.24	0.924	0.76	0.0076	0.3308	0.0009
0.047	0.05	20	0.01	1	1.0024	0.0007	0.9889	0.0006
0.047	0.05	20	0.06	0.9995	0.9856	0.0019	0.9565	0.0016
0.047	0.05	20	0.12	0.9922	0.934	0.0046	0.8564	0.0034
0.047	0.05	20	0.18	0.9653	0.8734	0.0053	0.5726	0.0016
0.047	0.05	20	0.24	0.9074	0.7613	0.0082	0.319	0.0009
0.029	0.03	5	0.01	1	1.0043	0.0007	1.0015	0.0006
0.029	0.03	5	0.06	0.9991	0.996	0.002	0.9815	0.0018
0.029	0.03	5	0.12	0.9875	0.9603	0.0049	0.9118	0.0042
0.029	0.03	5	0.18	0.95	0.9358	0.0063	0.7592	0.0032
0.029	0.03	5	0.24	0.8822	0.8662	0.0139	0.5705	0.0031
0.029	0.05	5	0.01	1	1.0008	0.0007	1	0.0006
0.029	0.05	5	0.06	1.0013	0.9923	0.002	0.9828	0.0018
0.029	0.05	5	0.12	1.012	0.9794	0.0053	0.9347	0.0044
0.029	0.05	5	0.18	1.0284	0.941	0.0068	0.8211	0.0038
0.029	0.05	5	0.24	1.0352	0.9181	0.017	0.6607	0.0044
0.029	0.07	5	0.01	1	1.0003	0.0007	1.0002	0.0007
0.029	0.07	5	0.06	1.0033	0.9939	0.002	0.9862	0.0018
0.029	0.07	5	0.12	1.0296	0.9928	0.006	0.9414	0.0045
0.029	0.07	5	0.18	1.0792	0.9468	0.0068	0.8603	0.0063
0.029	0.07	5	0.24	1.1326	0.9448	0.026	0.7225	0.0055
0.029	0.03	10	0.01	1	1.0047	0.0007	0.9973	0.0006
0.029	0.03	10	0.06	0.9988	0.9868	0.0019	0.9643	0.0016
0.029	0.03	10	0.12	0.9827	0.9437	0.0046	0.8818	0.0037
0.029	0.03	10	0.18	0.9297	0.901	0.0058	0.6269	0.0019
0.029	0.03	10	0.24	0.8325	0.7965	0.0087	0.383	0.0013
0.029	0.05	10	0.01	1	0.9999	0.0007	0.9953	0.0006
0.029	0.05	10	0.06	1.001	0.9912	0.002	0.9712	0.0017
0.029	0.05	10	0.12	1.008	0.9628	0.005	0.8987	0.0039
0.029	0.05	10	0.18	1.0127	0.9189	0.006	0.7106	0.0026
0.029	0.05	10	0.24	0.9969	0.8605	0.0108	0.4946	0.0021
0.029	0.07	10	0.01	1	1.0005	0.0007	0.9951	0.0006
0.029	0.07	10	0.06	1.003	0.9906	0.002	0.9774	0.0017

0.029	0.07	10	0.12	1.0262	0.9638	0.005	0.9214	0.004
0.029	0.07	10	0.18	1.0664	0.9172	0.0057	0.7464	0.0028
0.029	0.07	10	0.24	1.1011	0.8631	0.0097	0.5584	0.0027
0.029	0.03	20	0.01	1	1.0016	0.0007	0.9918	0.0006
0.029	0.03	20	0.06	0.9985	0.9852	0.0019	0.9448	0.0014
0.029	0.03	20	0.12	0.9795	0.9359	0.0045	0.8138	0.0028
0.029	0.03	20	0.18	0.9157	0.8203	0.0045	0.4813	0.001
0.029	0.03	20	0.24	0.7973	0.6632	0.0053	0.2416	0.0005
0.029	0.05	20	0.01	1	0.9997	0.0007	0.9896	0.0006
0.029	0.05	20	0.06	1.0008	0.9824	0.0019	0.9529	0.0015
0.029	0.05	20	0.12	1.0054	0.9385	0.0046	0.849	0.003
0.029	0.05	20	0.18	1.0021	0.8609	0.0048	0.577	0.0014
0.029	0.05	20	0.24	0.9701	0.7597	0.008	0.3447	0.0009
0.029	0.07	20	0.01	1	1.0009	0.0007	0.9904	0.0006
0.029	0.07	20	0.06	1.0028	0.9903	0.0019	0.958	0.0015
0.029	0.07	20	0.12	1.024	0.9596	0.0049	0.873	0.0032
0.029	0.07	20	0.18	1.0577	0.8955	0.0057	0.6183	0.0016
0.029	0.07	20	0.24	1.0793	0.7946	0.0096	0.3902	0.0011

Table A3. The effects of uncertainty, market power, demand growth and discount rate on κ. OLS results.

	Real options model	Quadratic adjustment costs	
		$b_q = 0.5$	$b_q = 3.0$
σ	-0.068	-0.709	-2.186
	(1.49)	(18.26)**	(22.51)**
η	-0.001	-0.003	-0.009
	(1.85)	(6.84)**	(7.00)**
r	2.253	0.755	1.647
	(6.68)**	(2.62)*	(2.28)*
μ	-1.847	-0.102	-0.166
	(3.92)**	(0.25)	(0.16)
Observations	102	102	102
R-squared	0.41	0.80	0.85

Note: Absolute values of t-statistics in parentheses. * significant at 5% level; ** significant at 1% level. A constant is included in all regressions.